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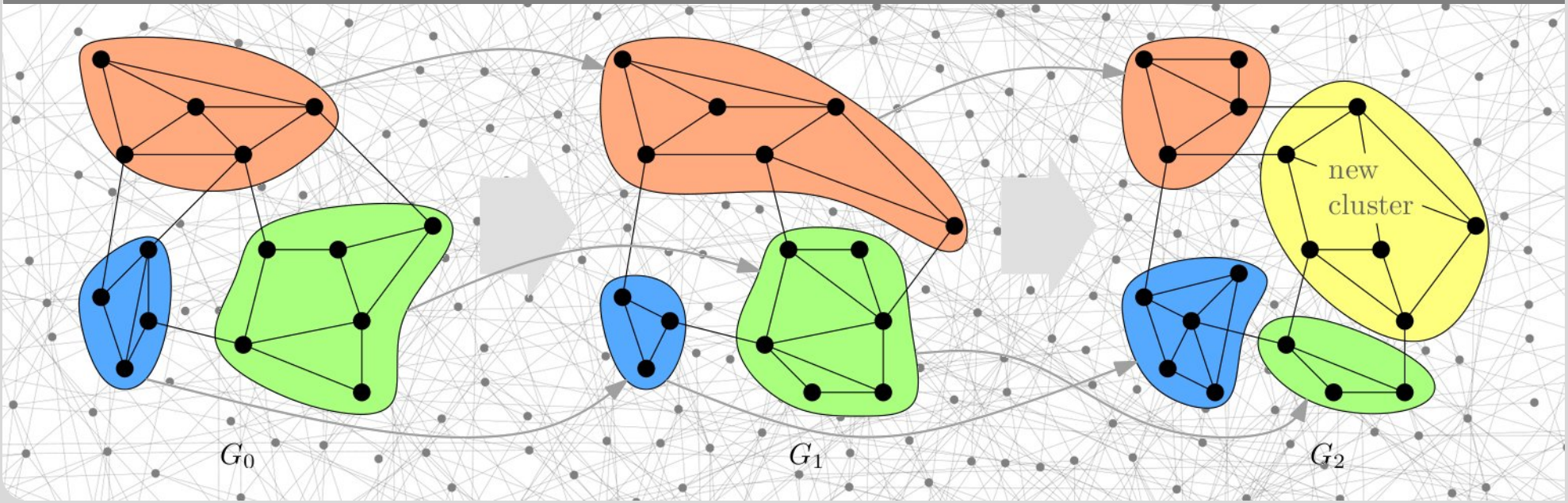
Modularity-Driven Clustering of Dynamic Graphs

SEA 2010 - 9th International Symposium on Experimental Algorithms

Robert Görke, Pascal Maillard, Christian Staudt, Dorothea Wagner | May 22, 2010

ITI WAGNER - KARLSRUHE INSTITUTE OF TECHNOLOGY



Structure

1 Basics

2 Static Algorithms

3 Dynamic Algorithms

4 Results

Overview

Modularity-Driven Clustering of Dynamic Graphs

In this work we

- pioneer **online dynamic modularity maximization**
- develop dynamizations of
  - the currently best heuristic algorithms
  - an optimal algorithm
- evaluate them on
  - dynamic clustered random graphs
  - dynamic real-world networks
- give sound recommendations for the choice of an algorithm

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Clustering: Intuition and Formalization

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Task

partition a graph into **natural groups**

⇓

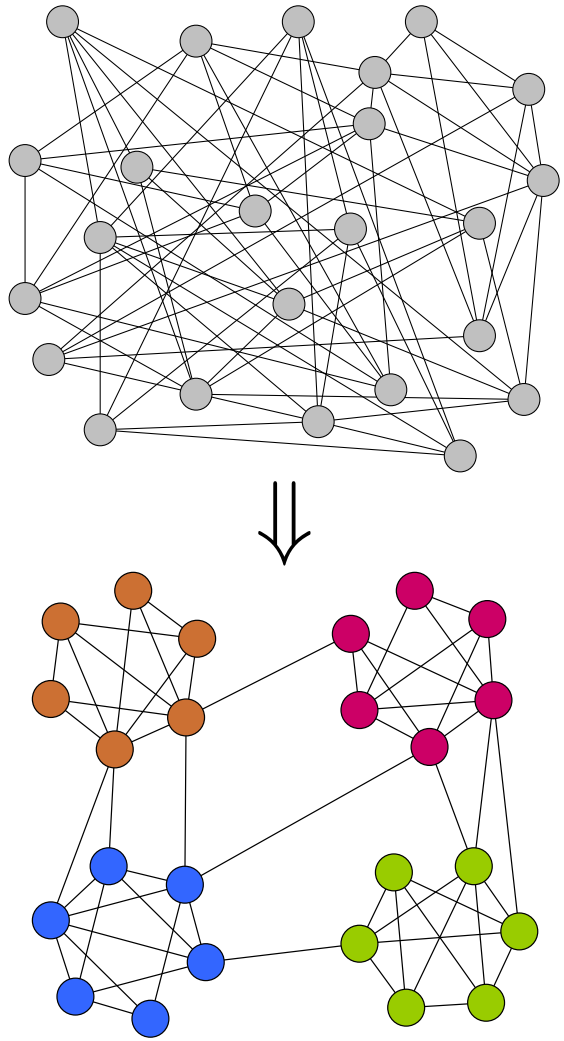
Paradigm

intra-cluster density vs.
inter-cluster sparsity

⇓

Formalization

a variety of **clustering quality indices**



Quality: Coverage & Modularity

- **Coverage**: fraction of intra-cluster edges
  - $cov(G, \mathcal{C}) \in [0, 1]$

Definition (Coverage)

$$cov(G, \mathcal{C}) := \sum_{C \in \mathcal{C}} \frac{|E(C)|}{|E|}$$

- **Modularity**: **Coverage** minus expected **coverage**
  - $mod(G, \mathcal{C}) \in [-1, 1]$ [GIRVAN, NEWMAN 2004]

Definition (Modularity)

$$mod(G, \mathcal{C}) := cov(G, \mathcal{C}(G)) - \mathbb{E}[cov(G, \mathcal{C}(G)]$$

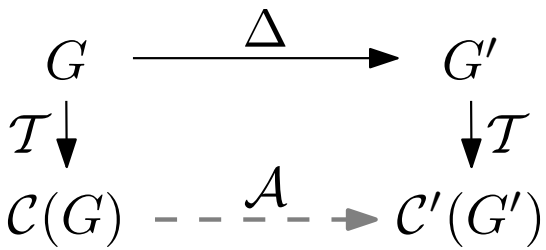
Dynamic Graph Clustering

Dynamic Instances

changing networks with evolving group structure

Dynamic Approach

update previous clustering reacting to changes in the graph



Clustering update problem

Criteria

- speed
- quality
- smooth transitions

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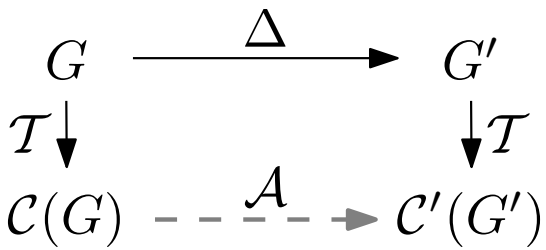

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Modularity: Optimization

Theorem

MODOPT is $\mathcal{NP}$-hard [BRANDES et al. 2008]

Corollary

DYNMODOPT is $\mathcal{NP}$-hard

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Heuristics: Global

[NEWMAN et al. 2004]

- globally greedy
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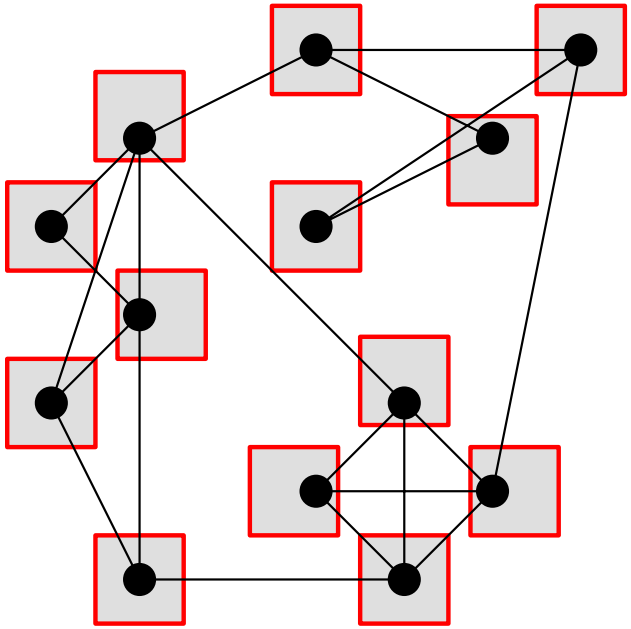
Algorithm 1: Global

1 **repeat**
2 | compute all $\Delta_{mod}(C_i, C_j)$
3 | merge best cluster pair
4 **until** *max.* $\Delta_{mod} \leq 0$

Illustration: Global

Dendrogram

Current Clustering



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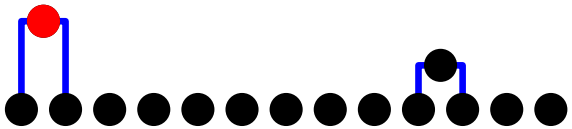

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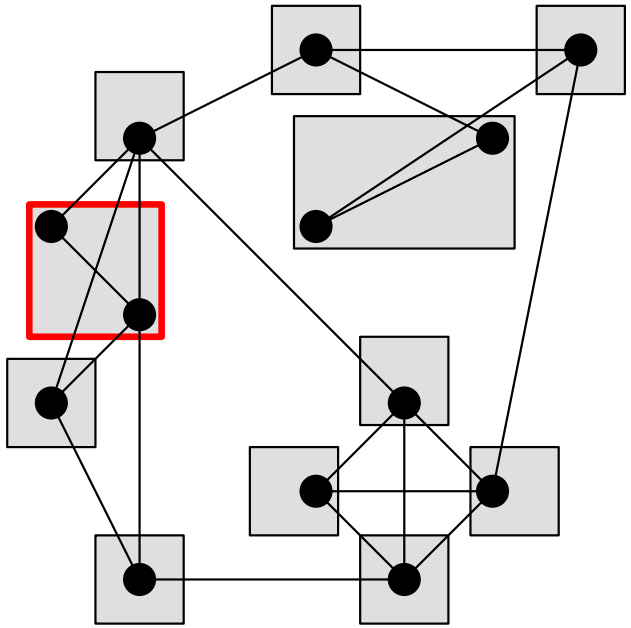

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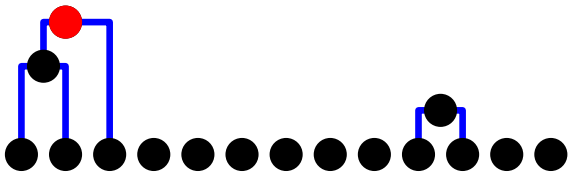

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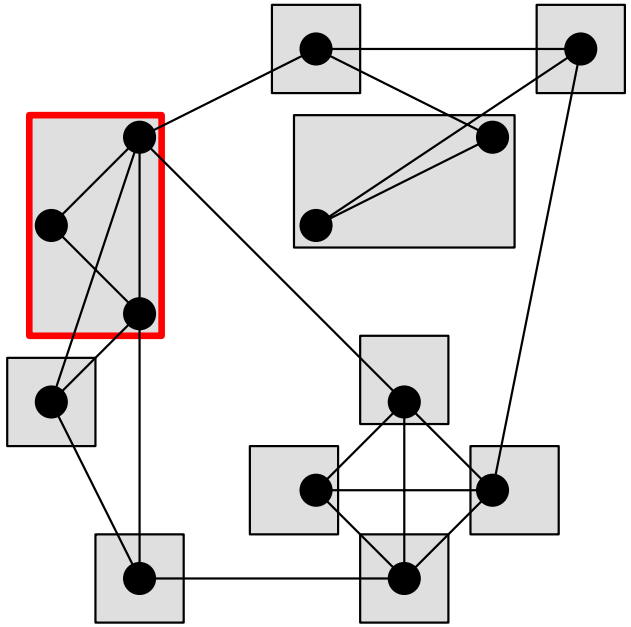

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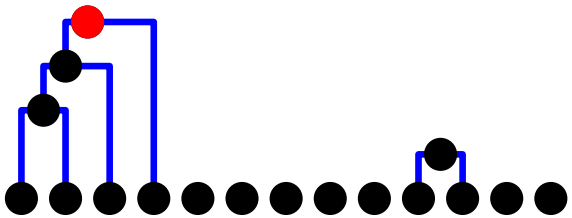

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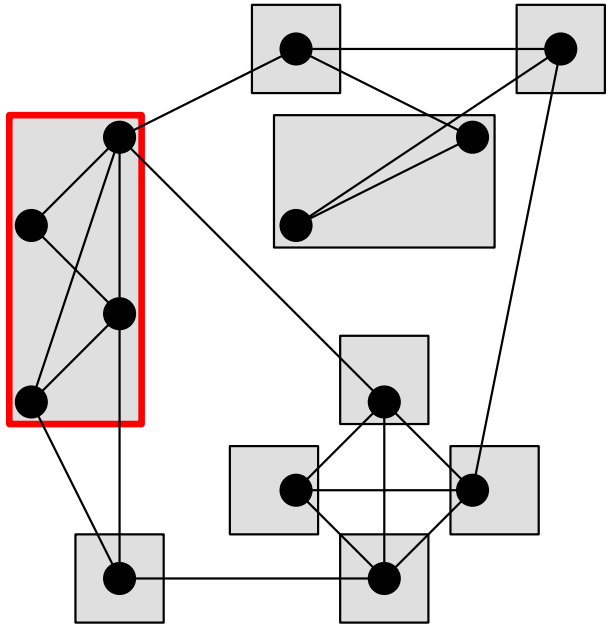

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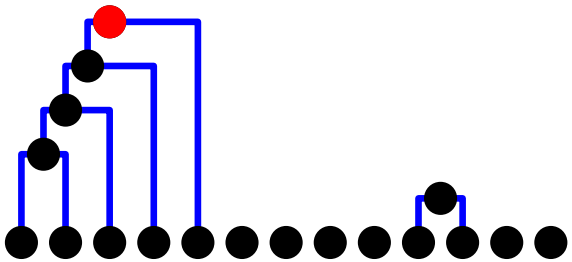

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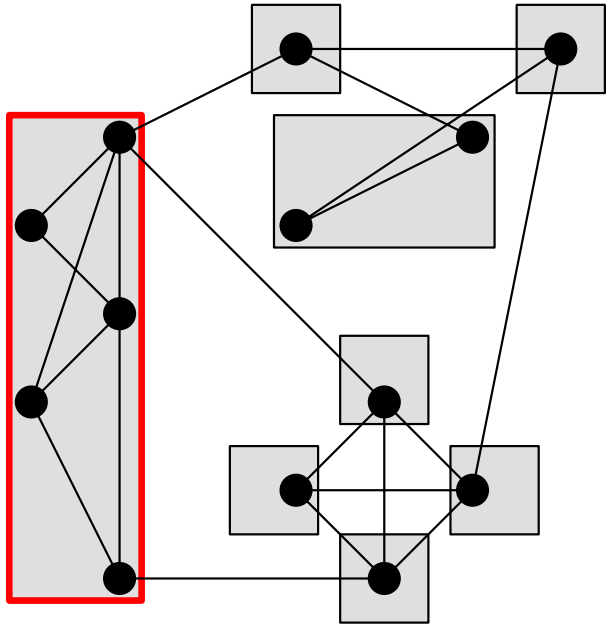

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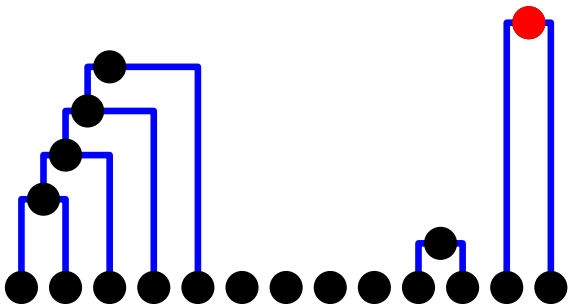

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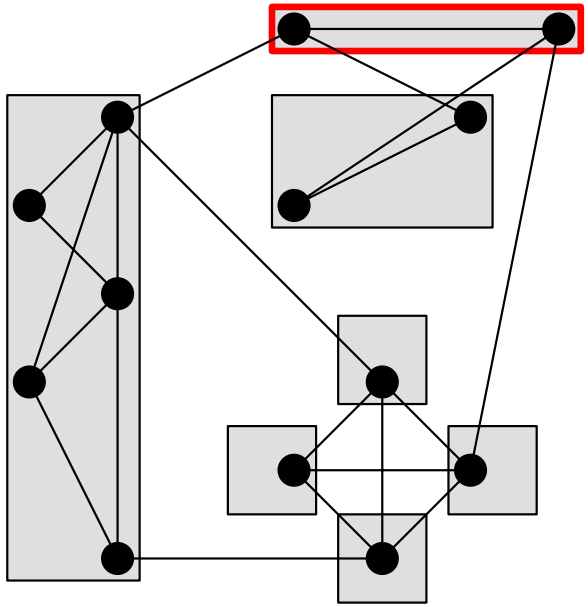

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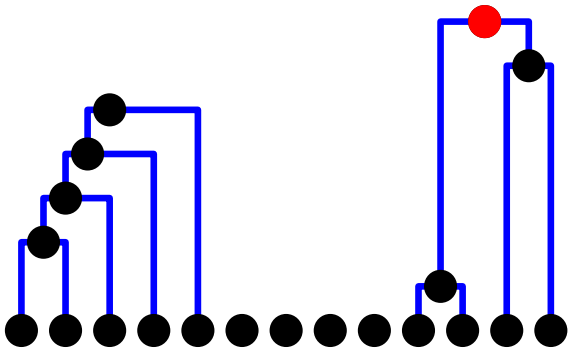

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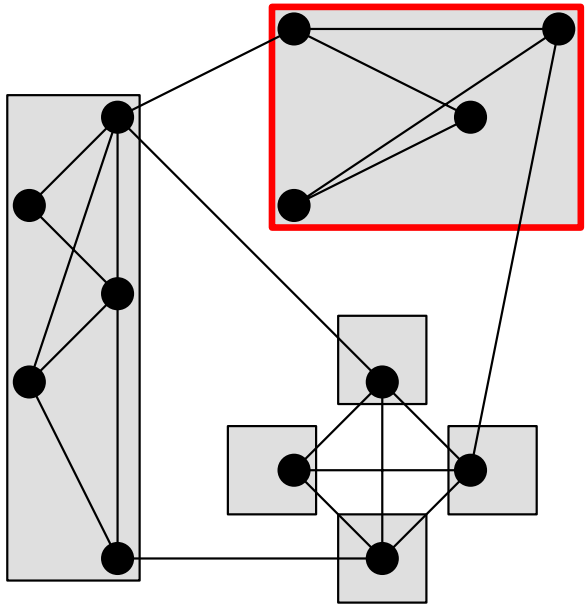

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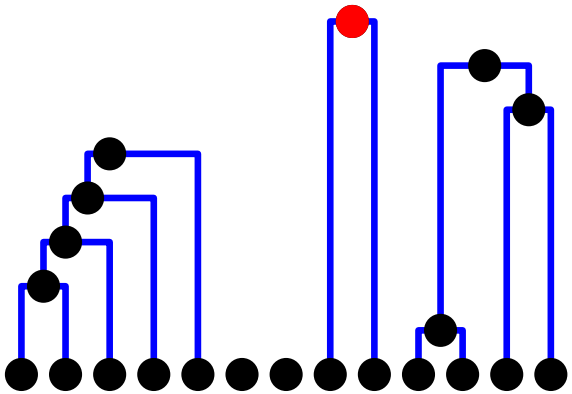

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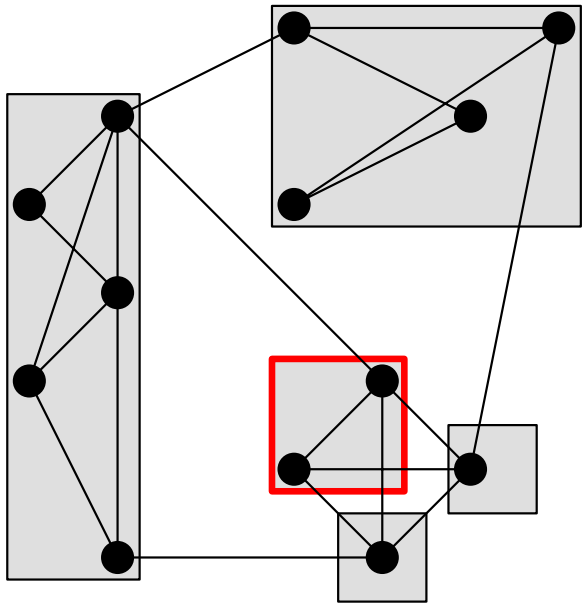

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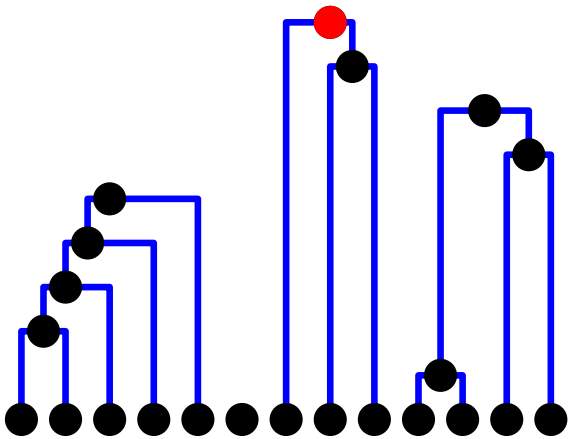

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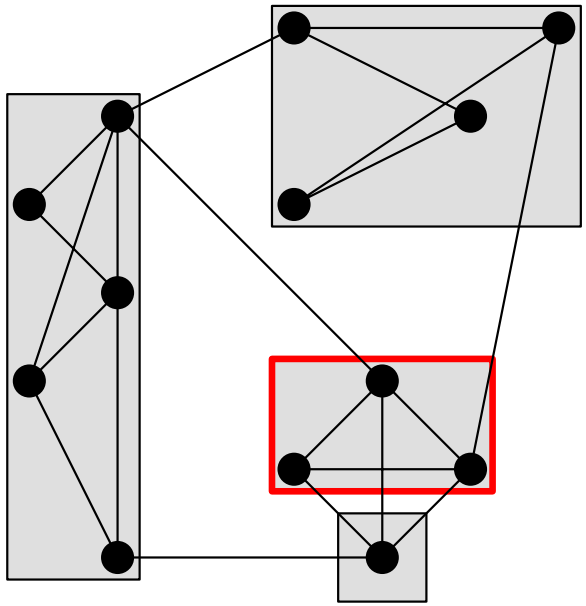

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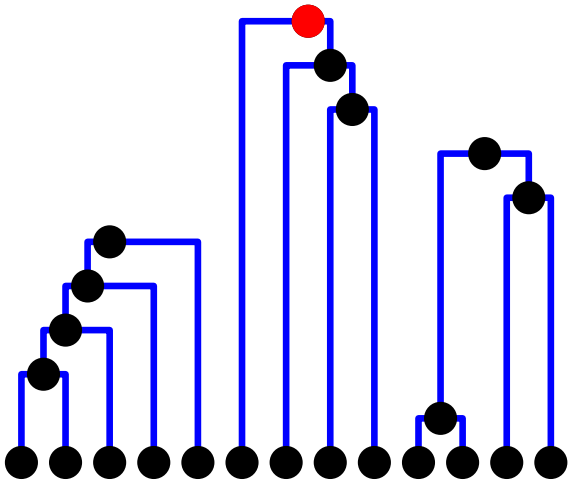

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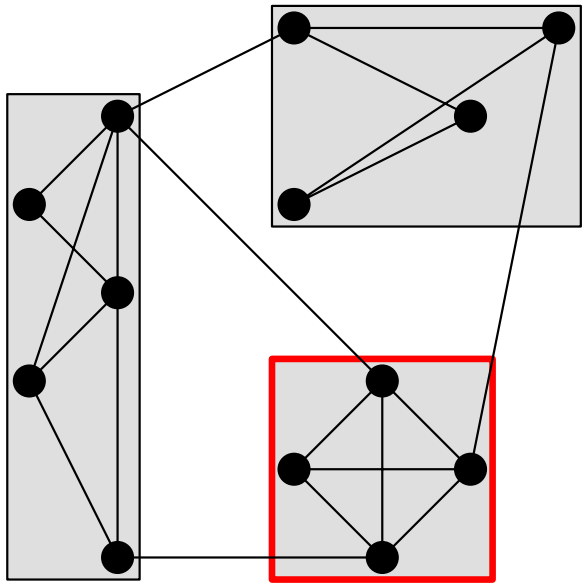

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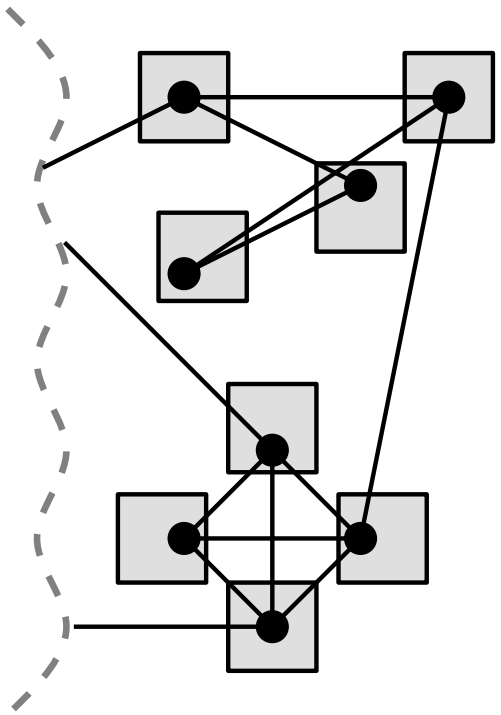

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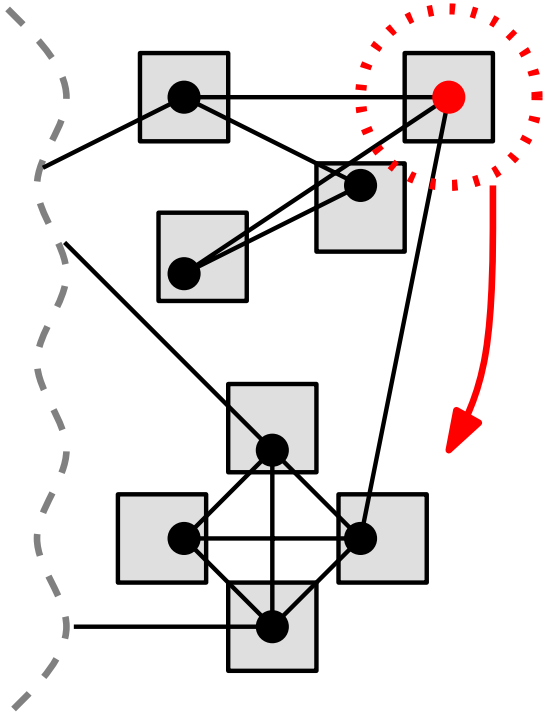

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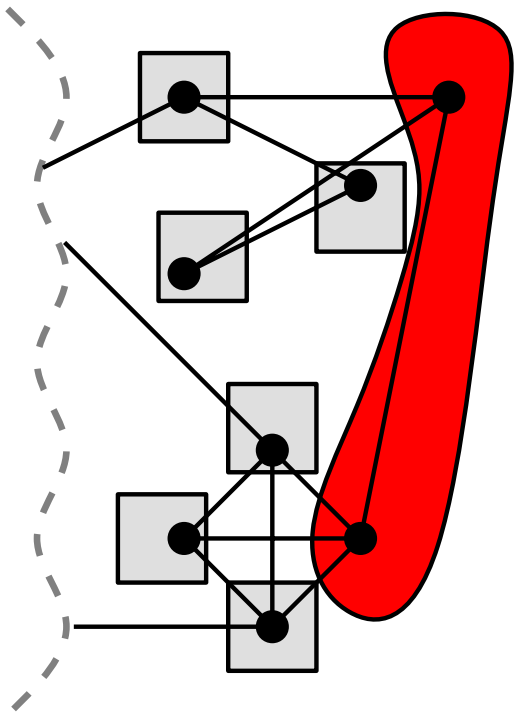

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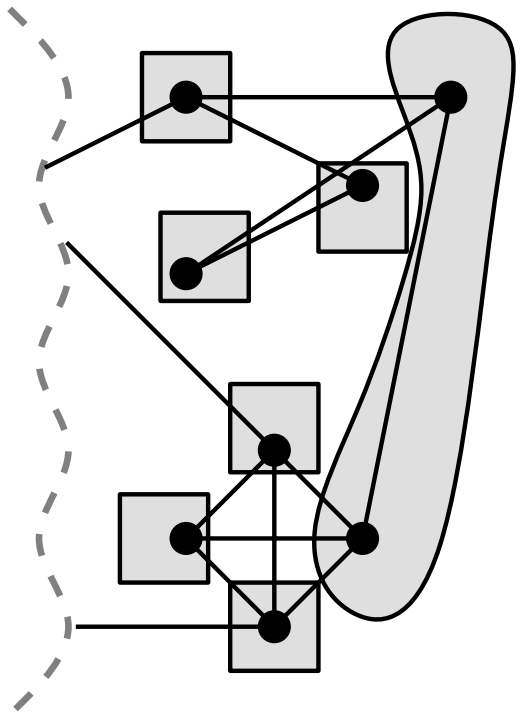

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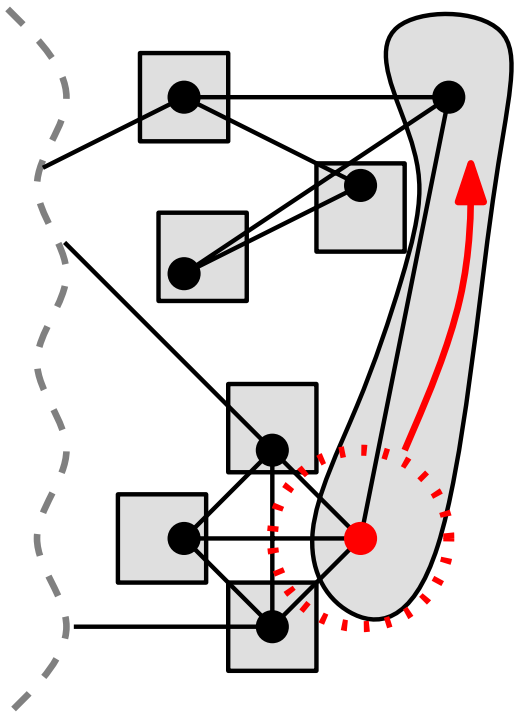

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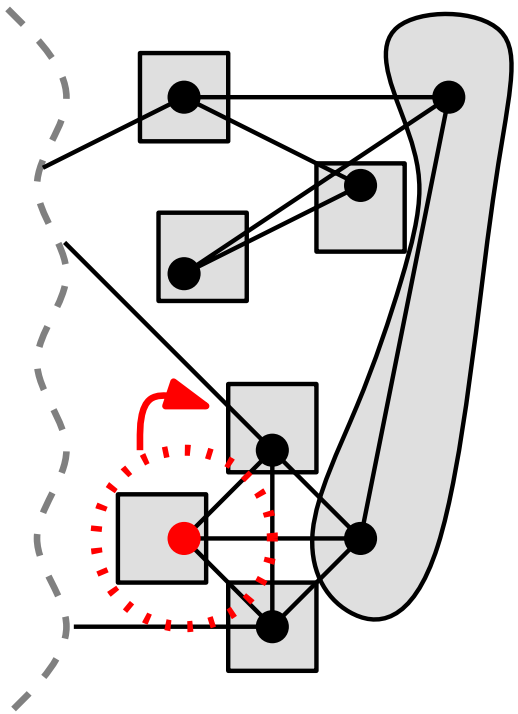

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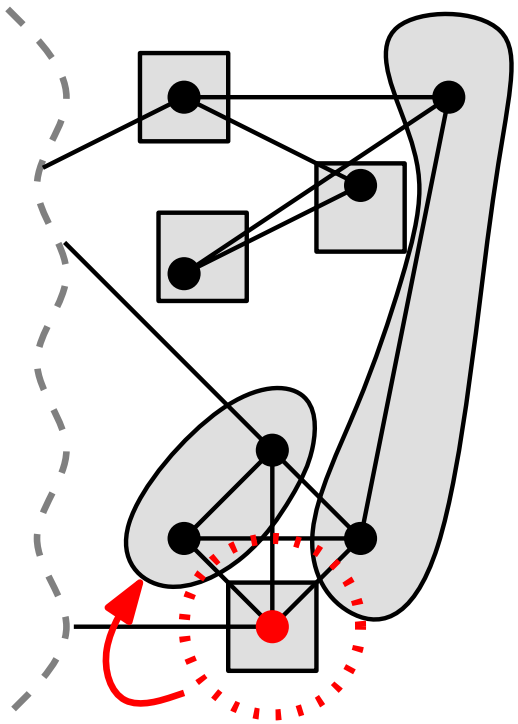

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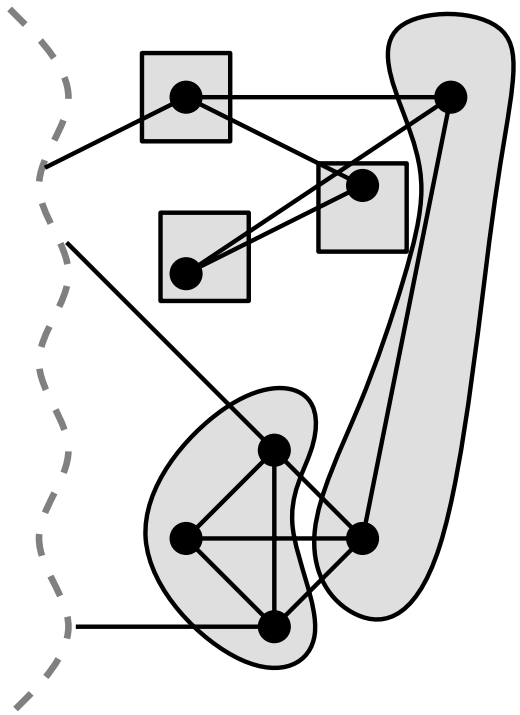

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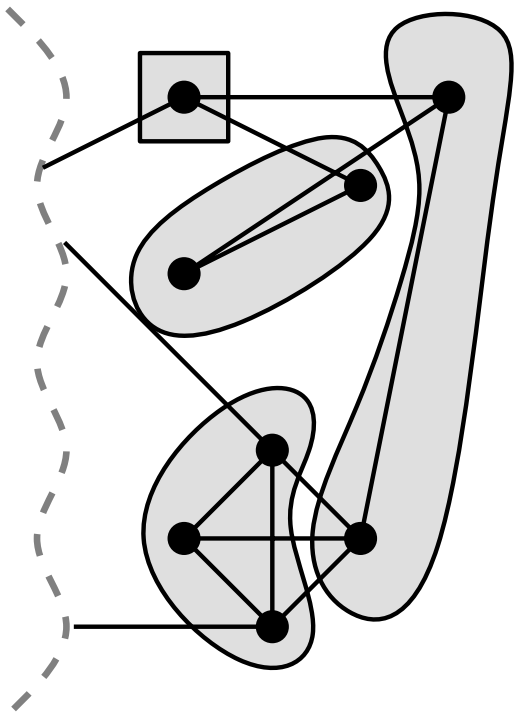

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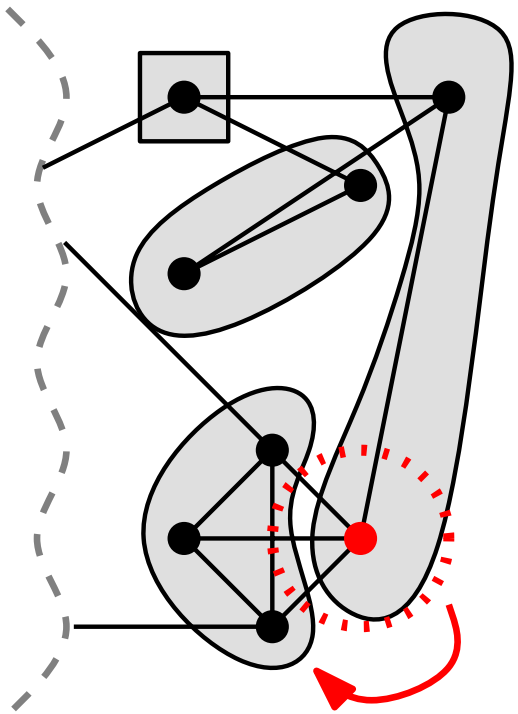

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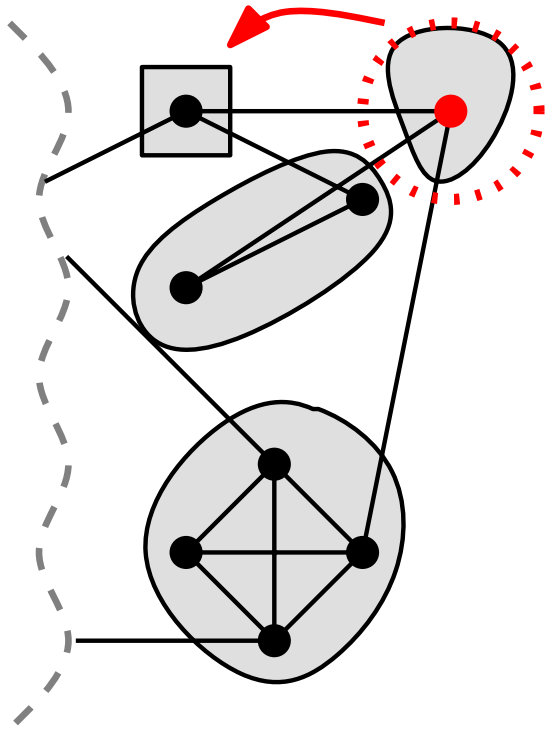

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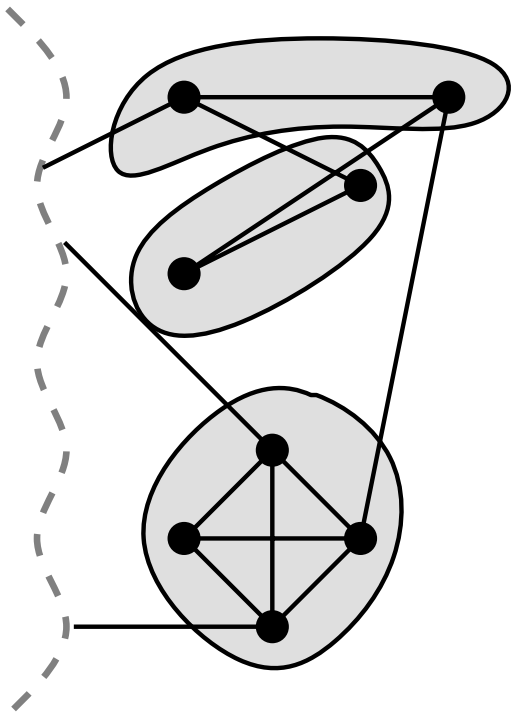

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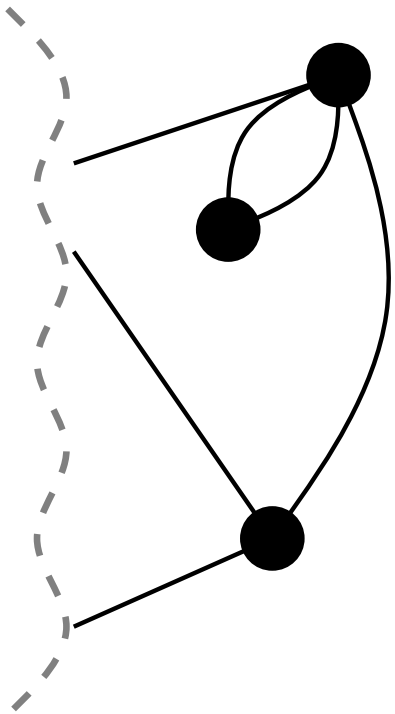

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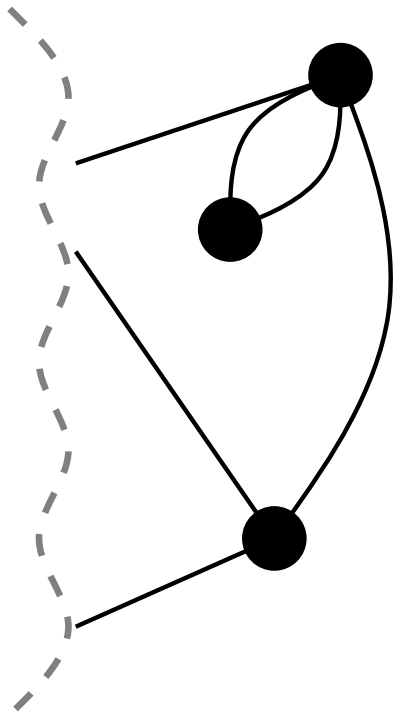

Pseudo-Dynamic Algorithm

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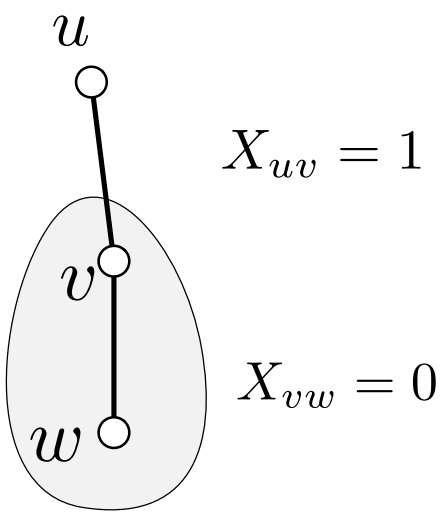
Optimization: ILP approach

1. introduce decision variables

$$\forall \{u, v\} \in \binom{V}{2} : X_{uv} = \begin{cases} 0 & \text{if } C(u) = C(v) \\ 1 & \text{otherwise} \end{cases}$$



2. ensure valid clustering with constraints (transitivity):

$$\forall \{u, v, w\} \in \binom{V}{3} : \begin{cases} X_{uv} + X_{vw} - X_{uw} \geq 0 \\ X_{uv} + X_{uw} - X_{vw} \geq 0 \\ X_{uw} + X_{vw} - X_{uv} \geq 0 \end{cases}$$

Optimization: ILP approach

3. optimize target function:

$$\mathrm{mod}_{\mathrm{ILP}}(G, \mathcal{C}_G) = \sum_{\{u,v\} \in \binom{V}{2}} \left(\omega(u,v) - \frac{\omega(u) \cdot \omega(v)}{2 \cdot \omega(E)} \right) \cdot X_{uv}$$

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Pseudo-Dynamic Algorithm

sILP: start **ILP** from scratch after each change. Infeasible!

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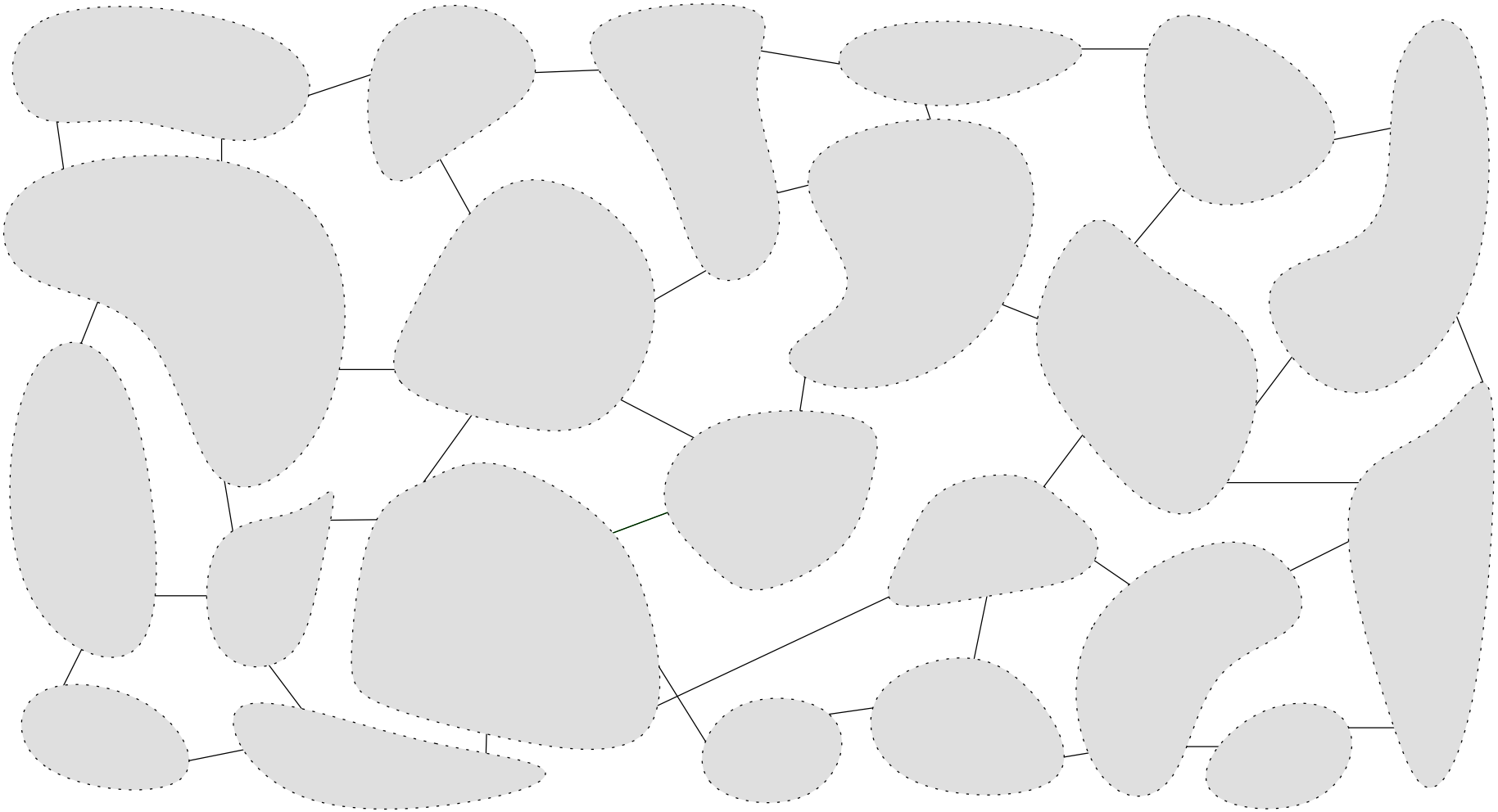

locality assumption

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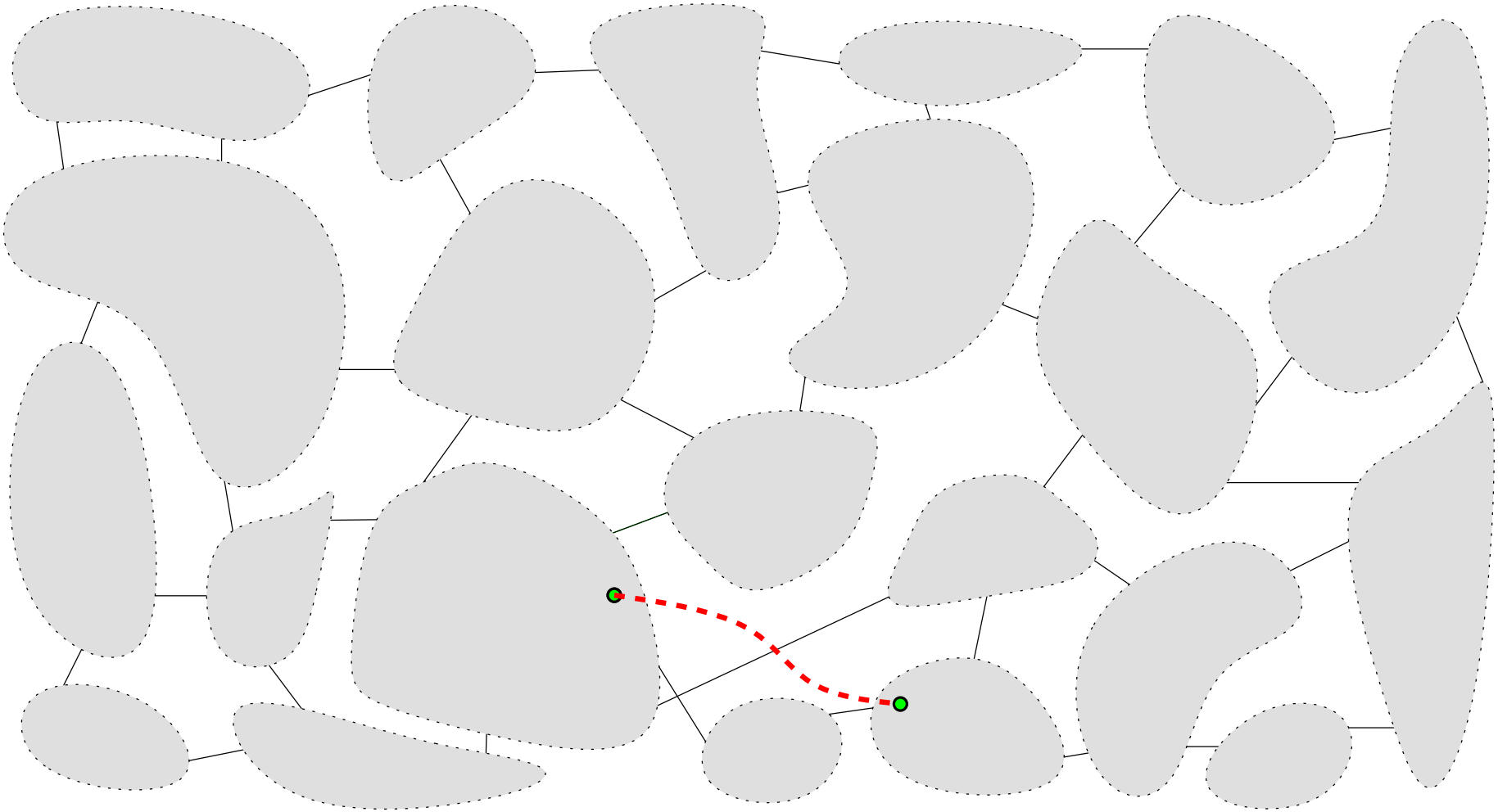

locality assumption

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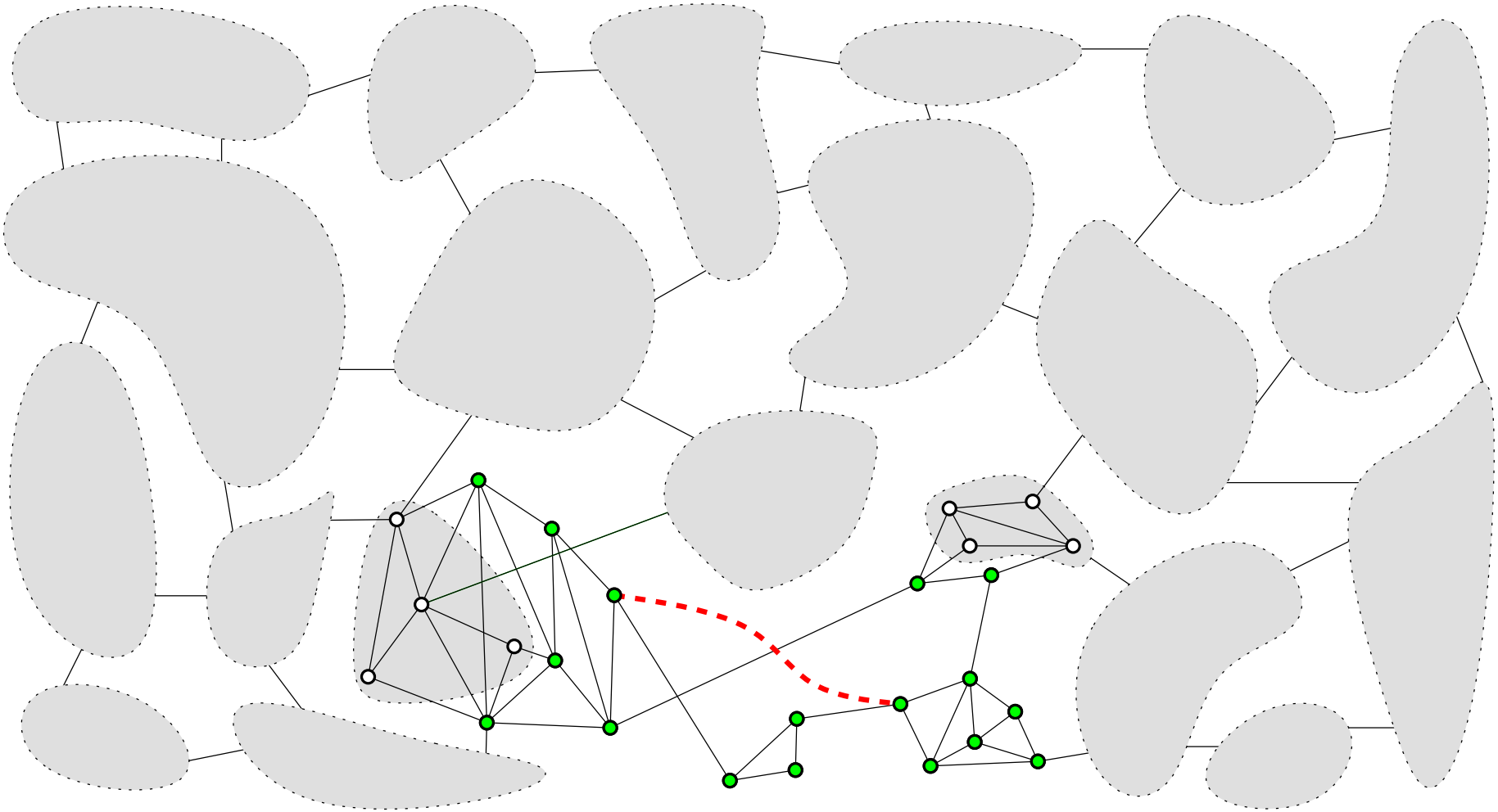

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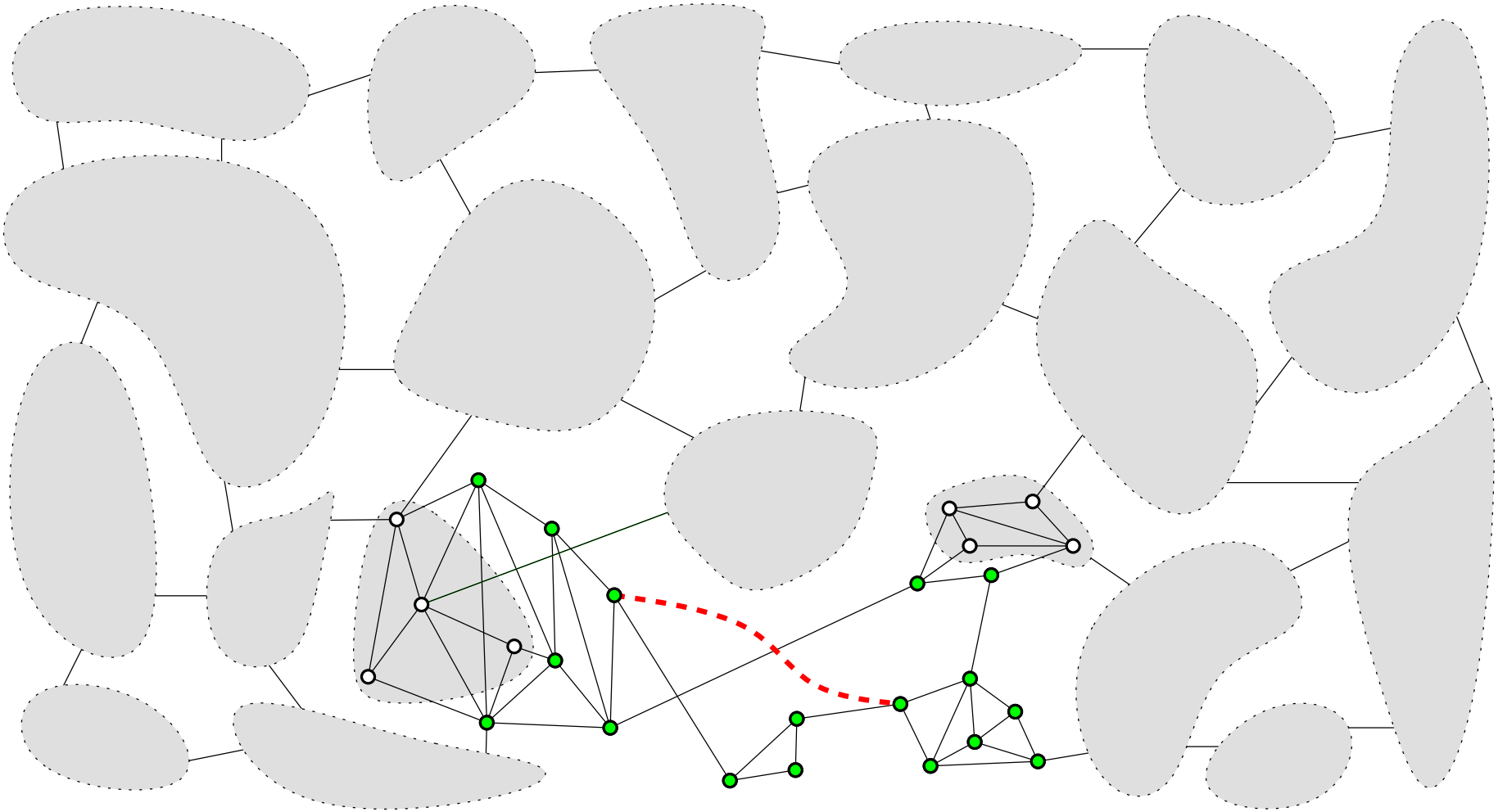
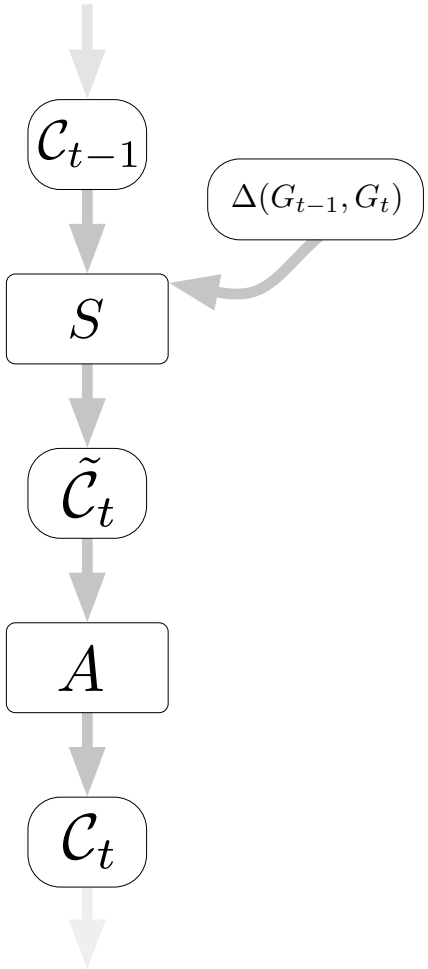


locality assumption

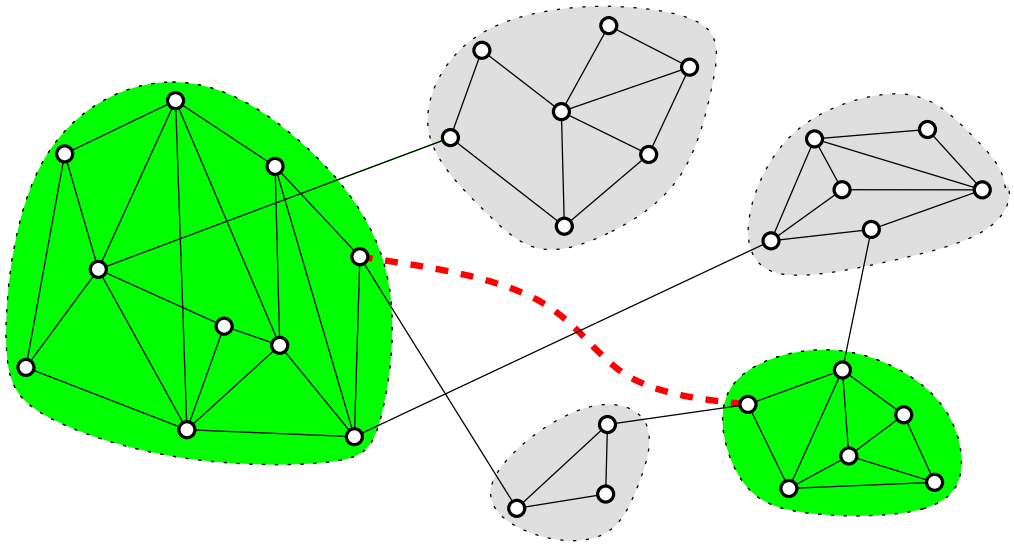
Prep Strategies: Concept

prep strategy S

- reacts to changes
- prepares half-finished **preclustering** $\tilde{\mathcal{C}}$
- passes $\tilde{\mathcal{C}}$ on to algorithm



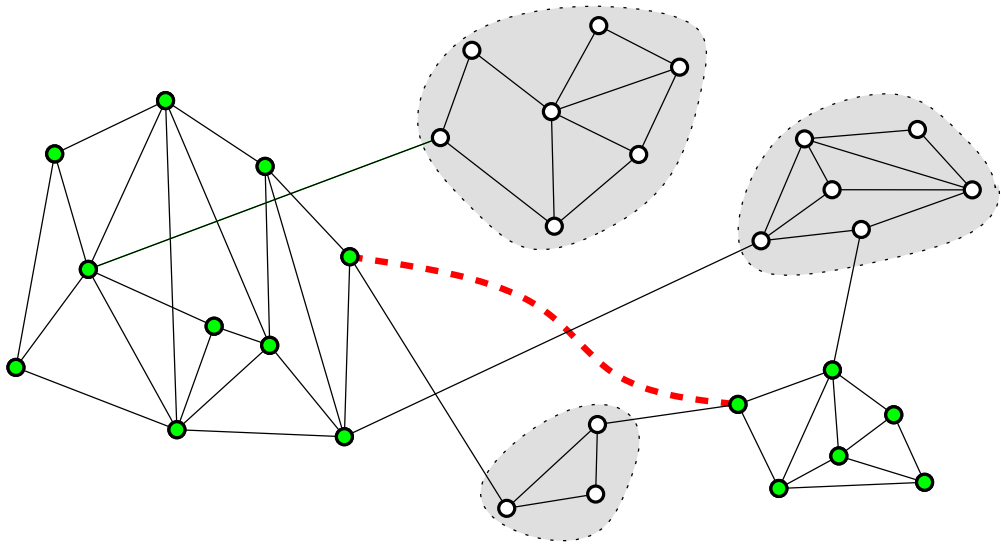
Prep Strategies: BU



Prep Strategy

BU: Break up the affected clusters entirely

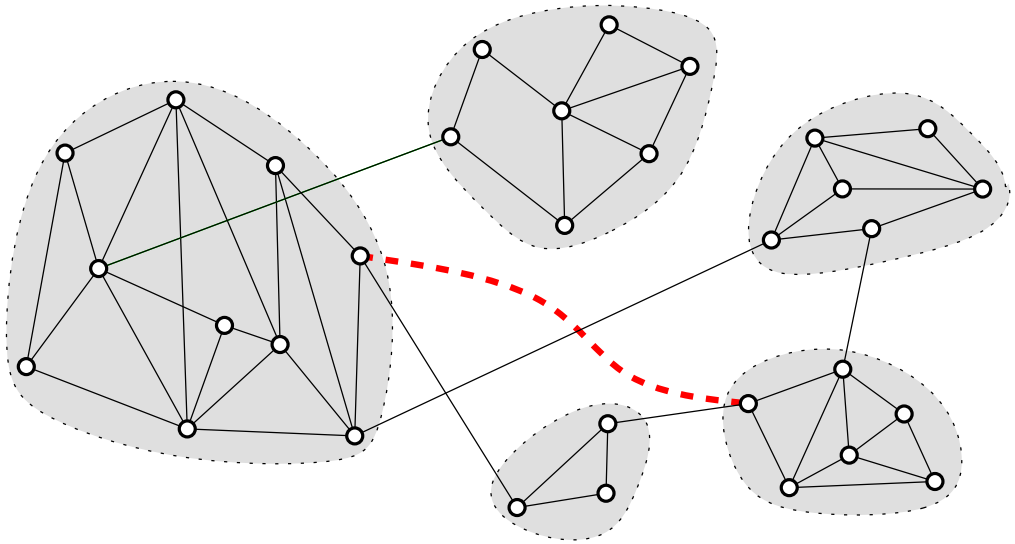
Prep Strategies: BU



Prep Strategy

BU: Break up the affected clusters entirely

Prep Strategies: N/BN

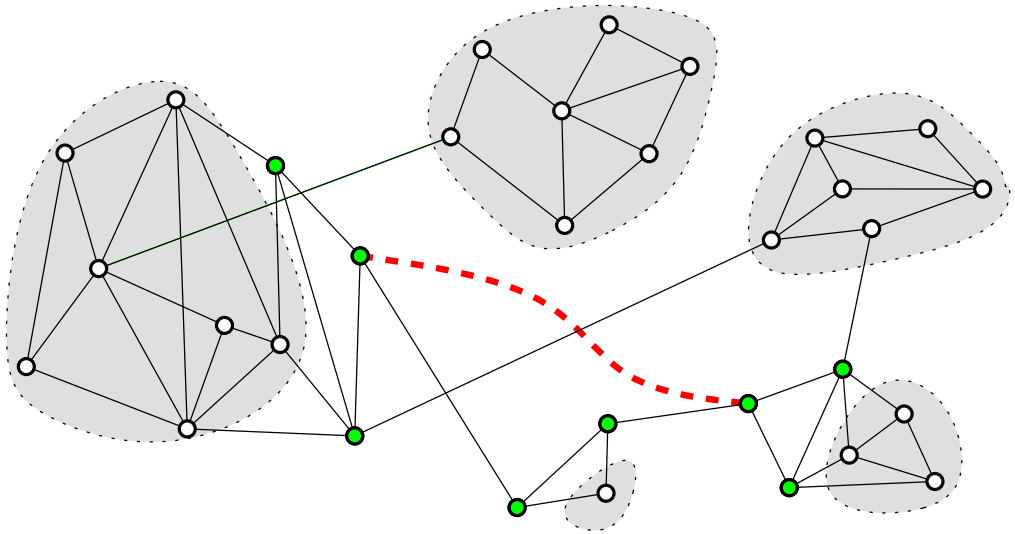


Prep Strategy

N: free a neighborhood up to a depth *d* with **BFS**

Experiments: What is the best choice for *d* / *k*?

Prep Strategies: N/BN



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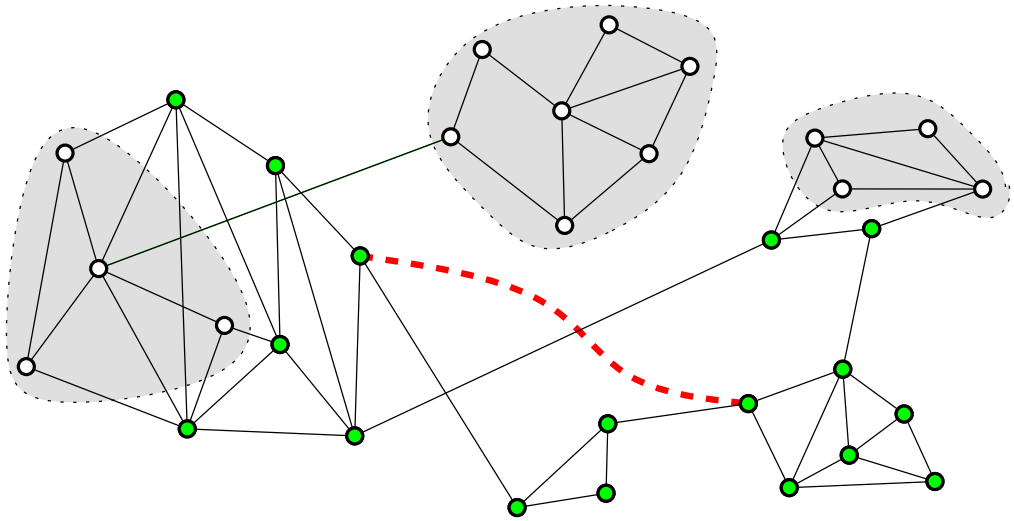

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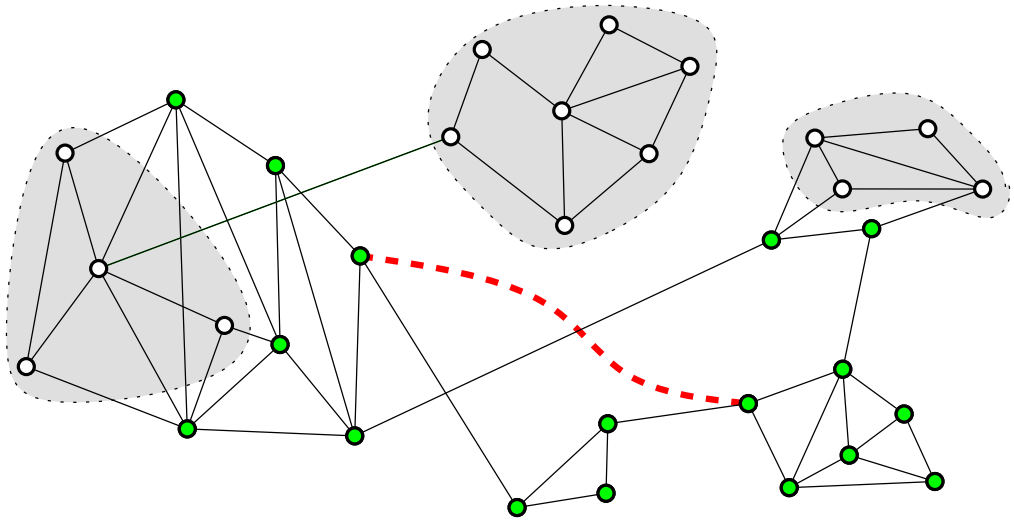

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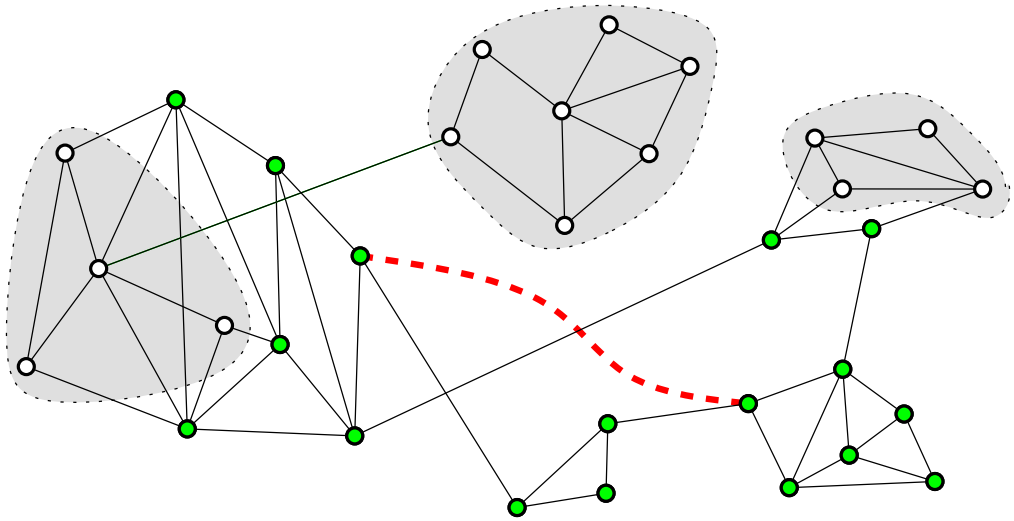

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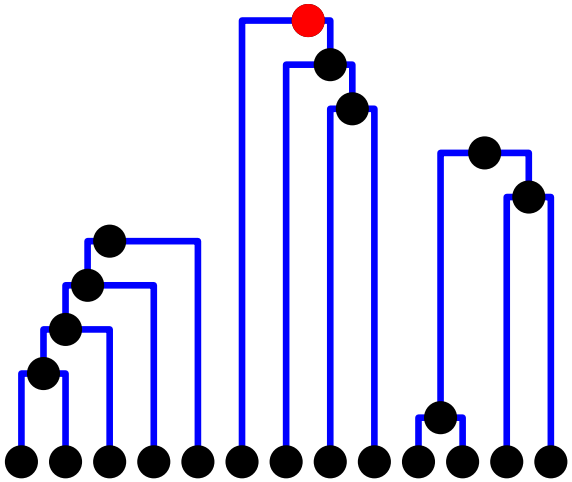
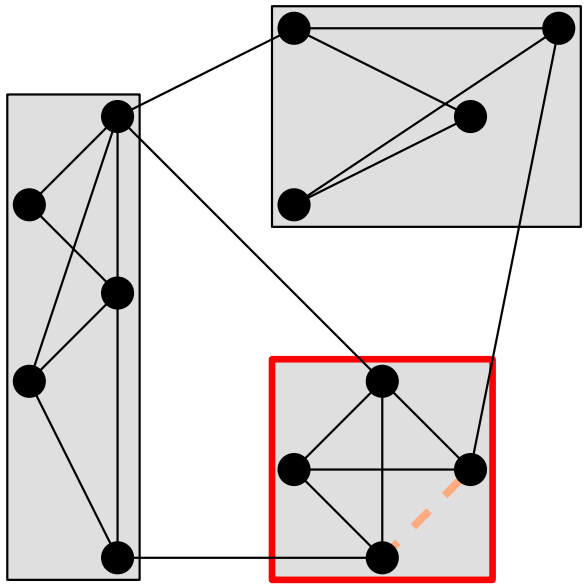
Experiments: What is the best choice for d / k?

Prep Strategy BT: Illustration

dendrogram



current clustering

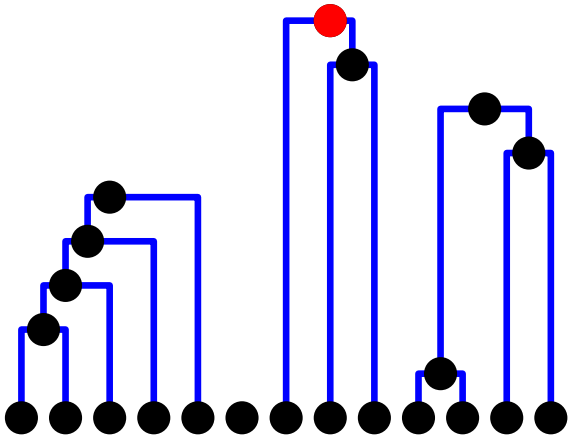


Prep Strategy

BT: Backtrack Global's mergers according to heuristic rules

Prep Strategy BT: Illustration

dendrogram



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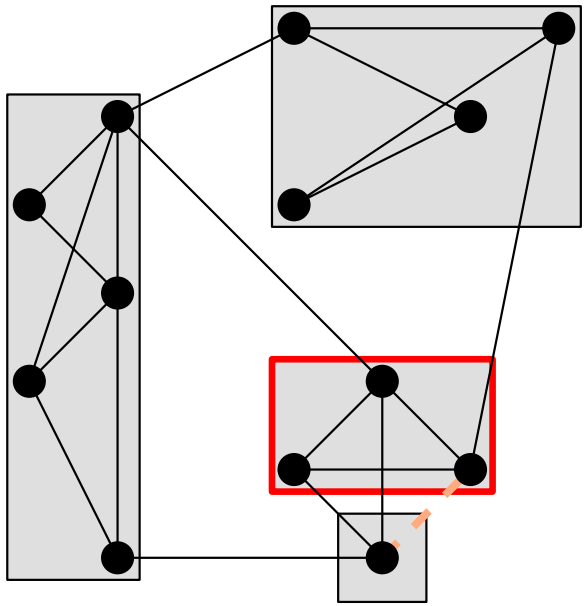

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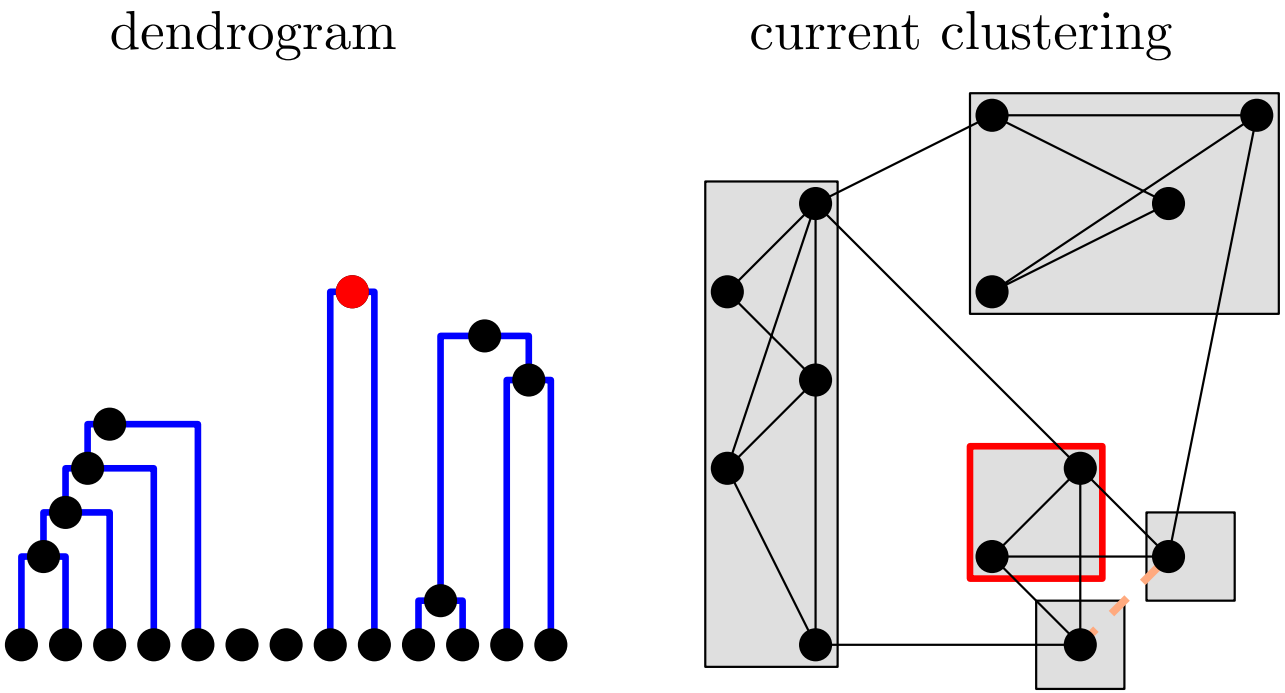

Prep Strategy

BT: Backtrack **Global**'s mergers according to heuristic rules

Dynamic Heuristics: dGlobal

1. take **preclustering** defined by **prep strategy**
2. run **Global** to complete

Dynamic Algorithm

dGlobal: Global with $\tilde{\mathcal{C}}$ as search space

Dynamic Heuristics: dGlobal

1. take **preclustering** defined by **prep strategy**
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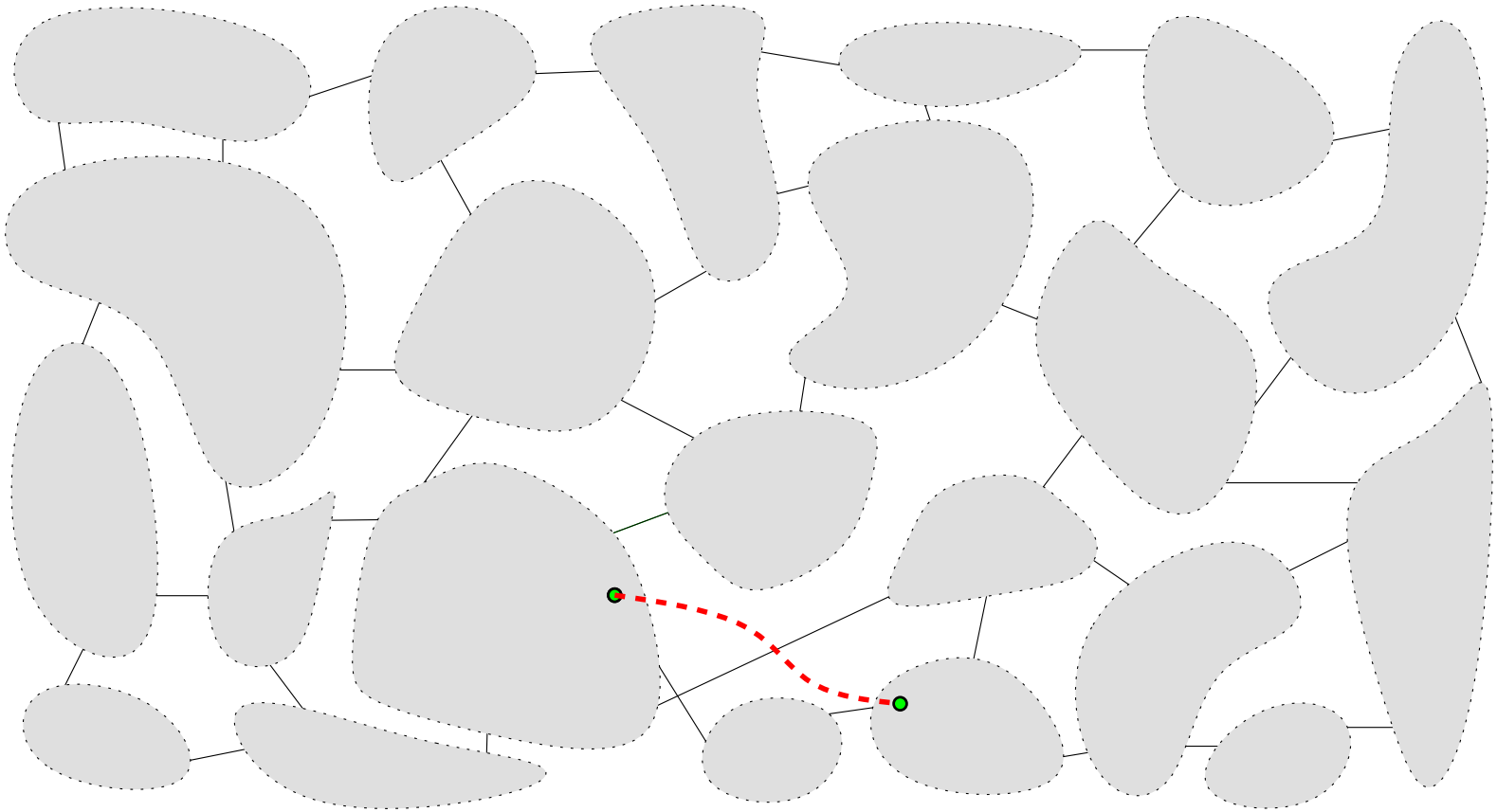
Dynamic Algorithm

dGlobal: **Global** with $\tilde{\mathcal{C}}$ as search space

Illustration: dLocal



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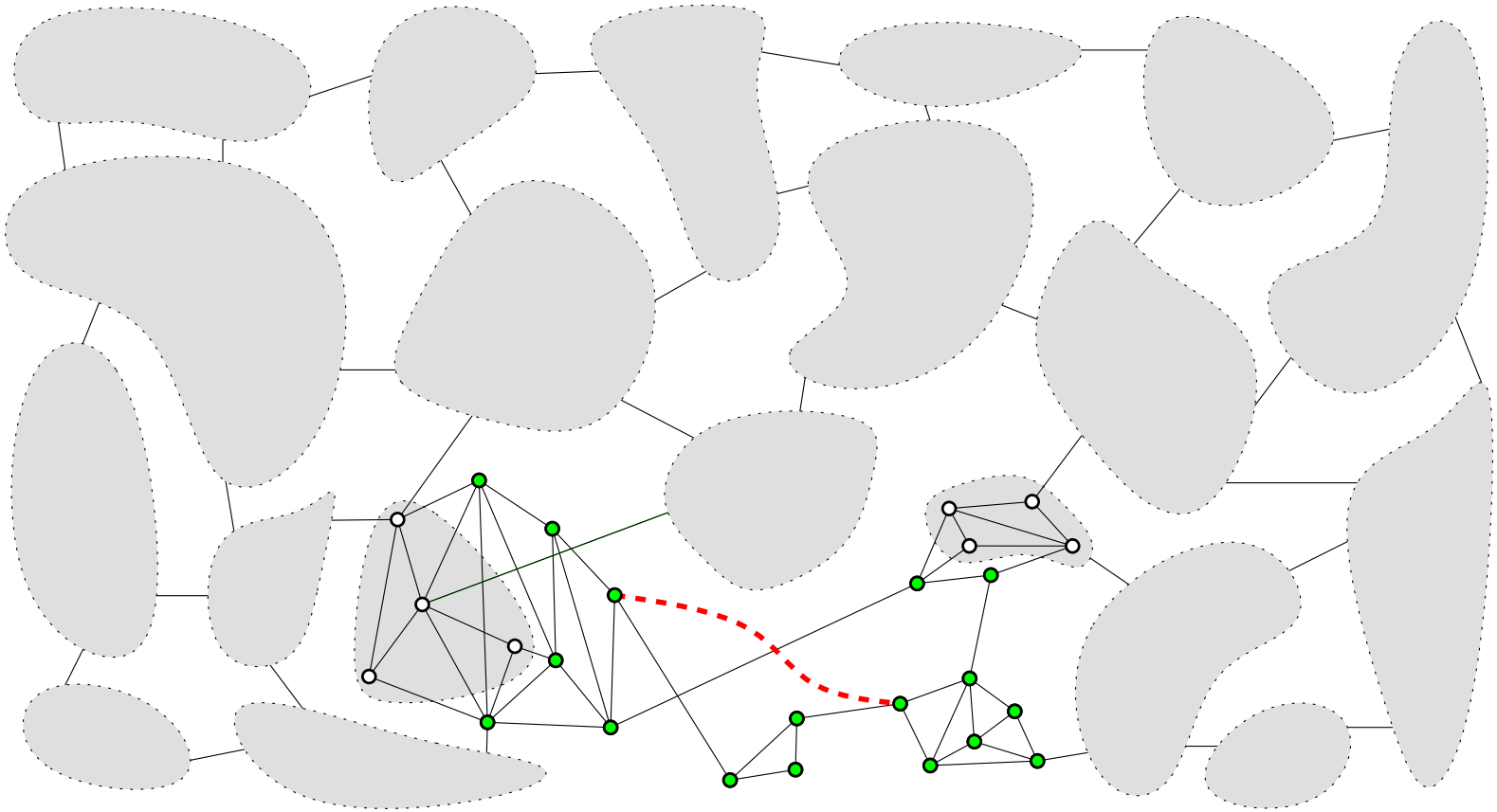

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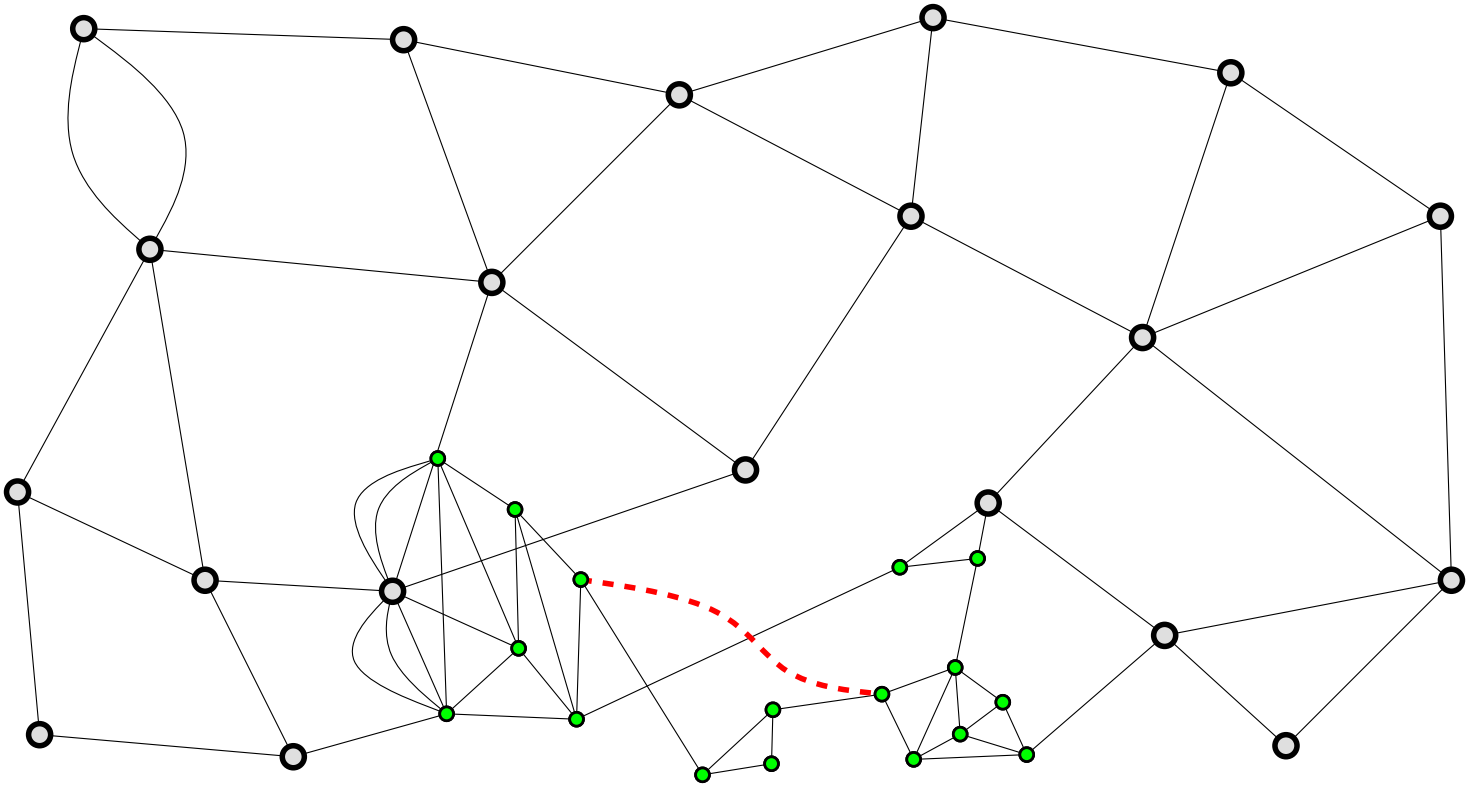

Dynamic Heuristics: dLocal

1. extract some nodes defined by **prep strategy** from **supernodes**
2. run **Local** to complete

Dynamic Algorithm

dLocal: **Local** with $\tilde{\mathcal{C}}$ as search space

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Dynamic Algorithm

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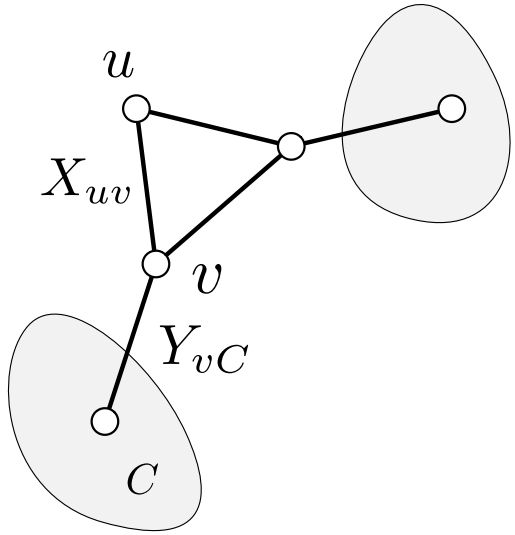
Dynamic Algorithms: dILP

1. construct smaller instance of the ILP using the **preclustering**
2. solve ILP



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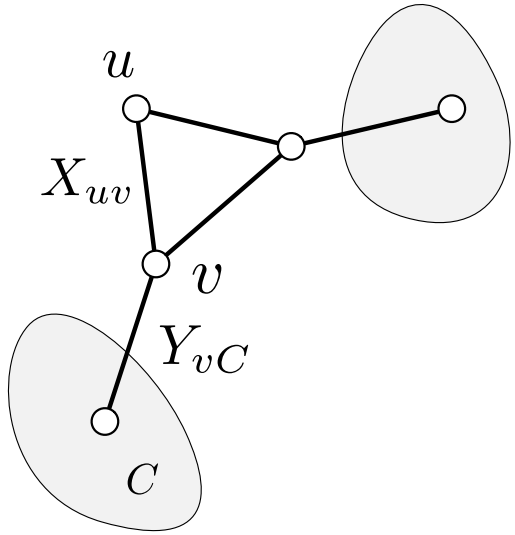


Dynamic Algorithm

dILP: **ILP** with $\tilde{\mathcal{C}}$ as search space

Structure

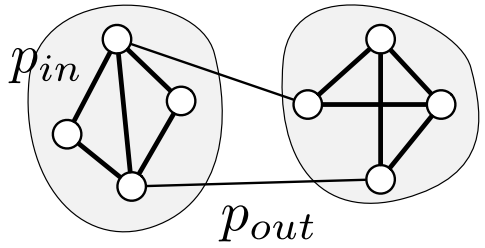
1 Basics

2 Static Algorithms

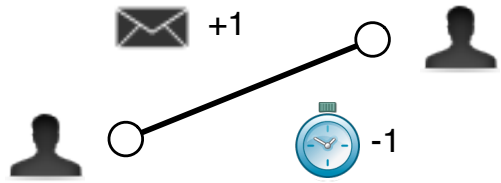
3 Dynamic Algorithms

4 Results

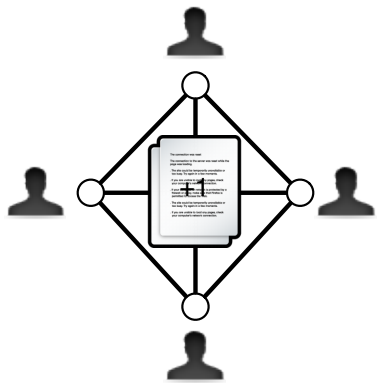
Dynamic Graph Instances



- generator for dynamic clustered random graphs [GÖRKE, STAUDT 2009]

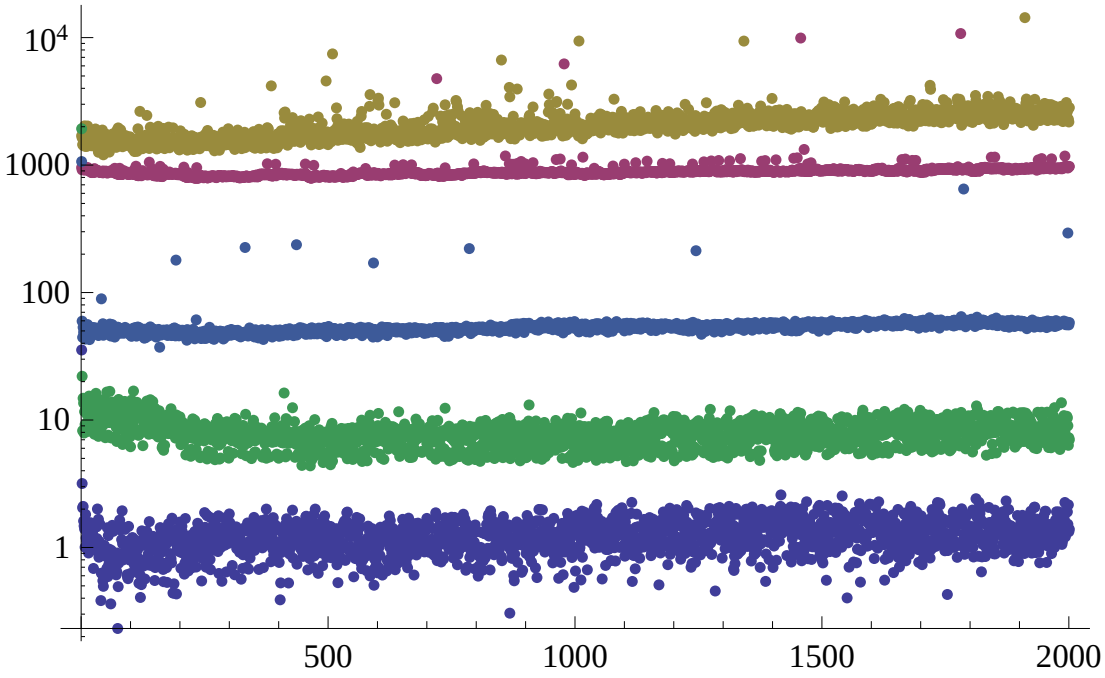


- e-mail graph of KIT CompSci



- arXiv collaboration graph

Dynamics vs Statics: Speed



running time [ms] per time step

sLocal
sGlobal

dGlobal@BN$_{16}$

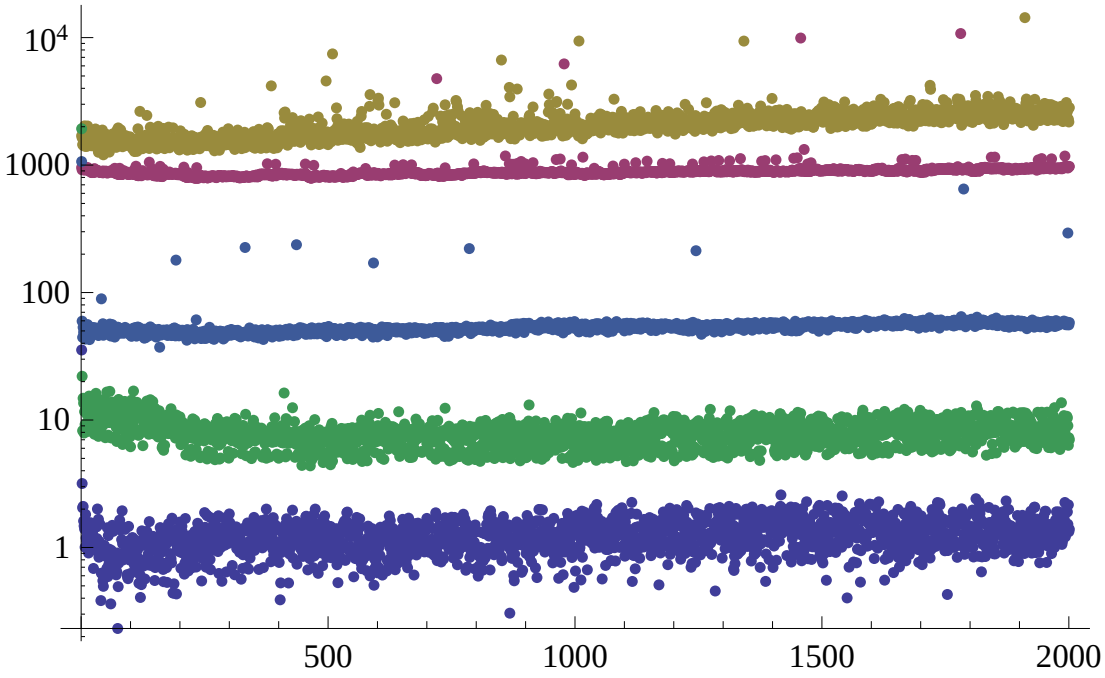
dLocal@BN$_4$
dGlobal@BT

Small search spaces work best!

Result

Dynamics run faster

Dynamics vs Statics: Speed



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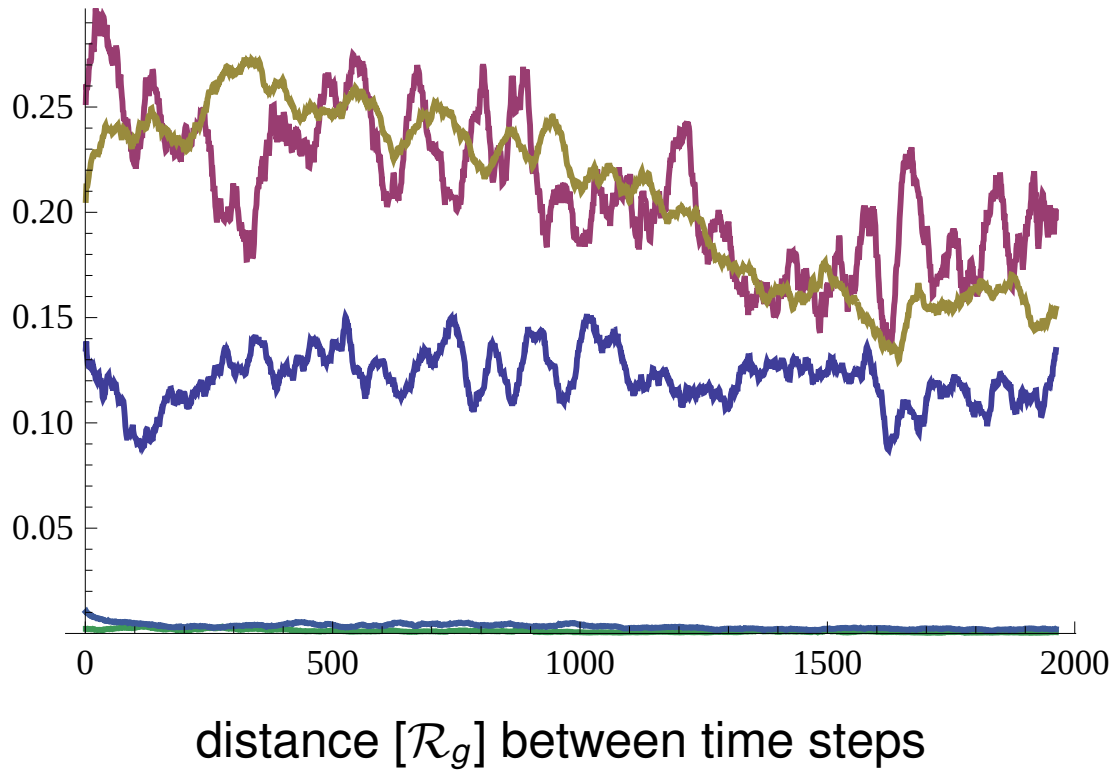
Dynamics vs Statics: Transitions



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sLocal

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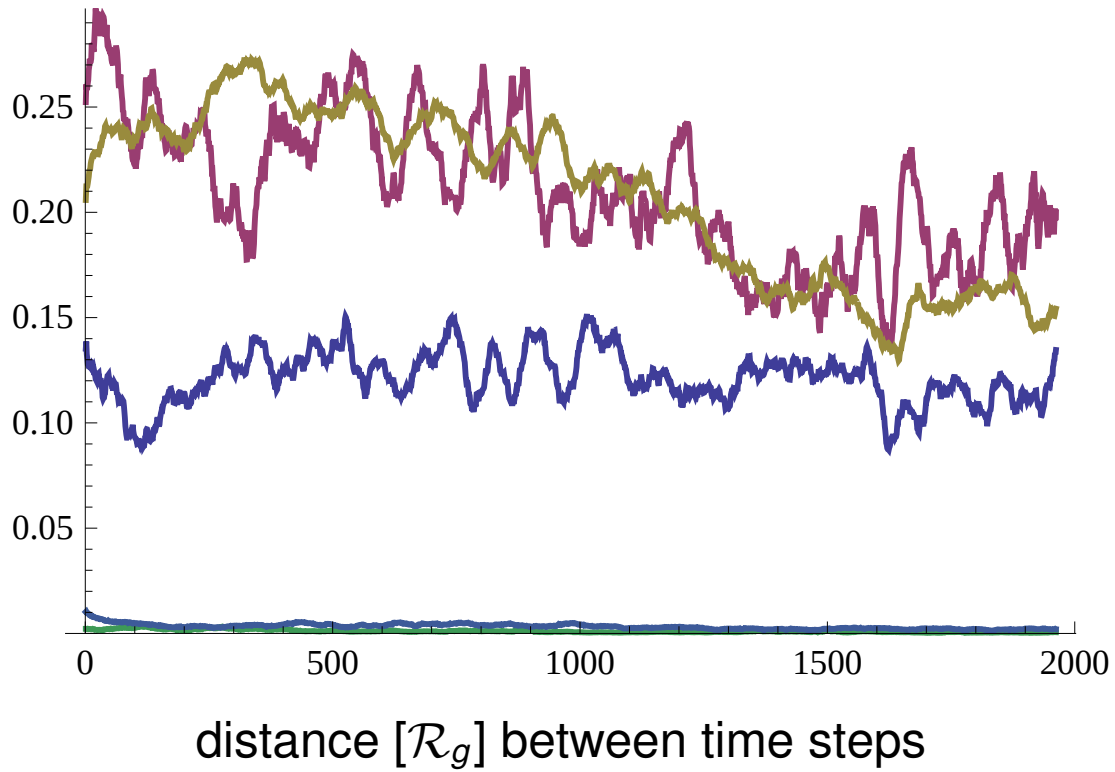
Result

Dynamics yield smoother clustering transitions

Dynamics vs Statics: Transitions



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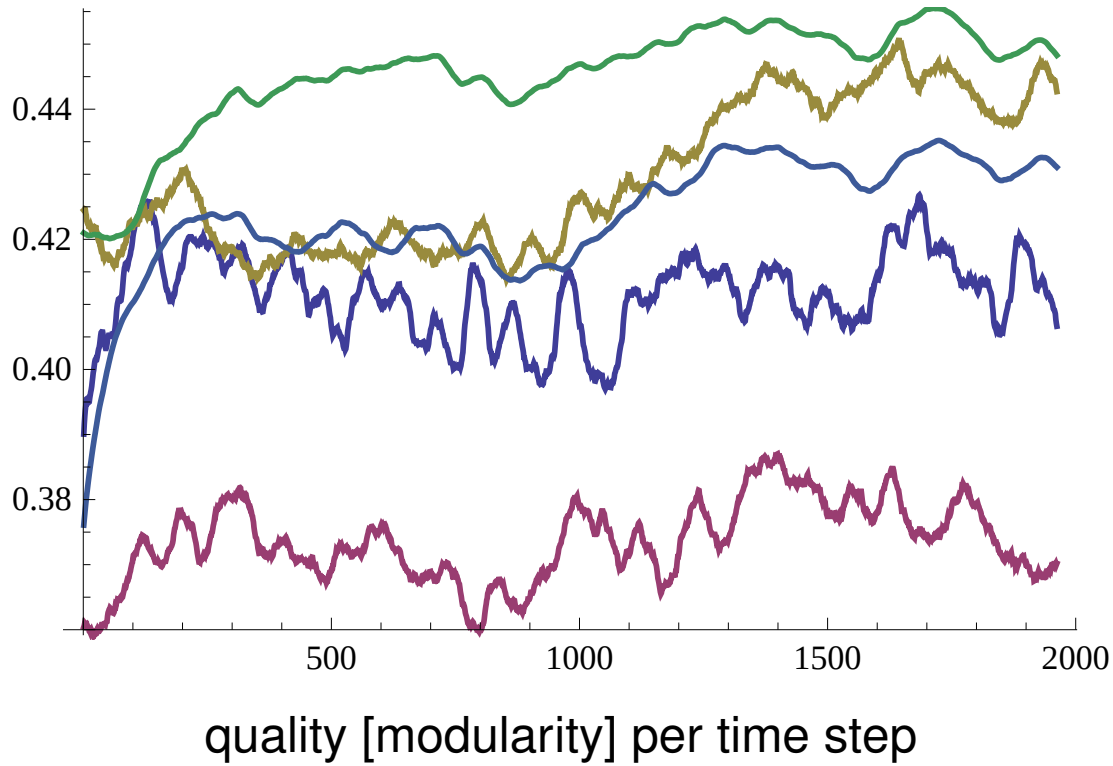
Dynamics vs Statics: Quality



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sLocal
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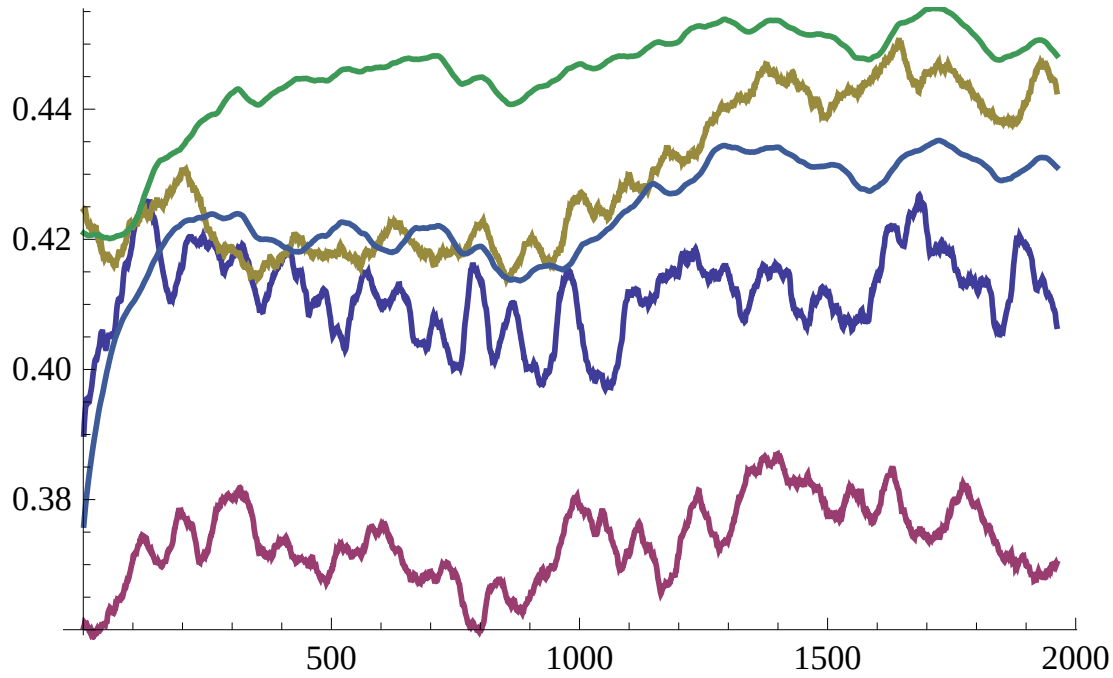
sGlobal

Result

Dynamics often surpass static counterparts in terms of quality

Dynamics vs Statics: Quality



quality [modularity] per time step

dLocal@BN_4
sLocal
dGlobal@BN_{16}

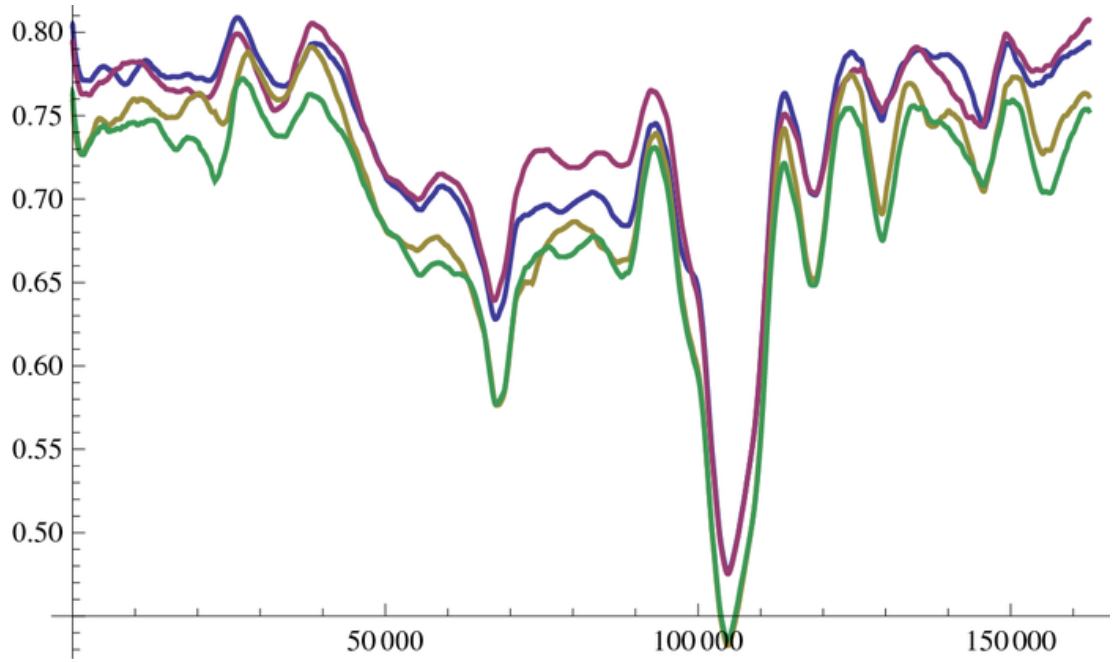
dGlobal@BT

sGlobal

Result

Dynamics often surpass static counterparts in terms of quality

Heuristics vs dILP: Quality



quality [modularity] per time step; e-mail graph

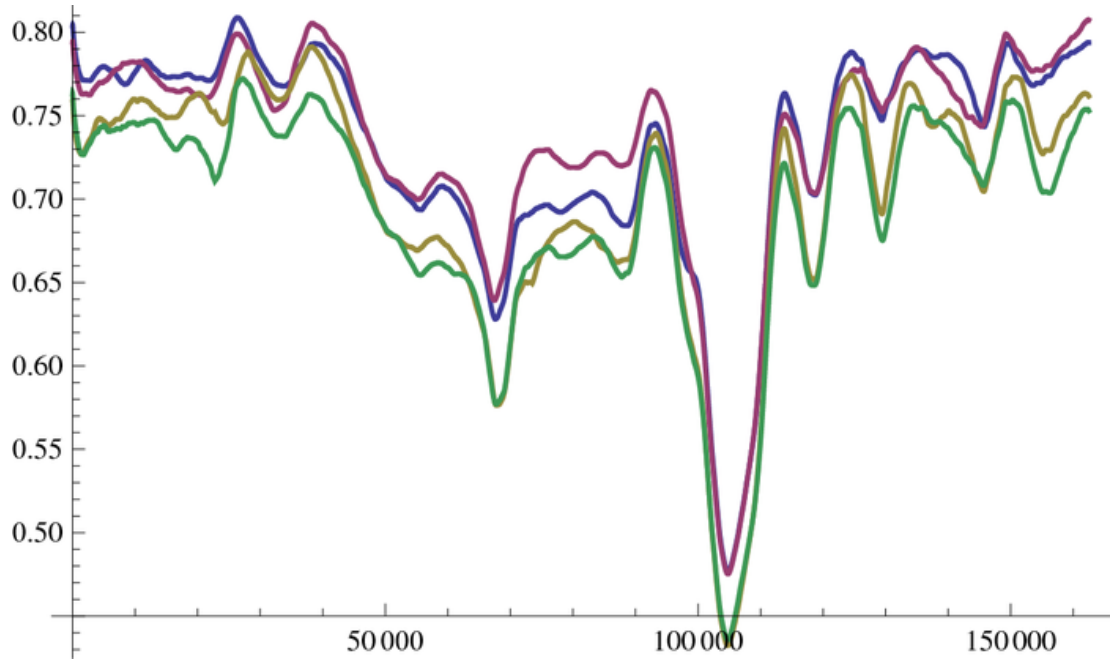
dGlobal@BN_8
dLocal@BN_8

dILP(var.1)@BN_8
dILP(var.2)@BN_8

Result

Local optimality is worse than dynamic heuristics.

Heuristics vs dILP: Quality



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- dynamic versions of state-of-the-art heuristics
- evaluation: dynamics yield better
  - speed
  - quality
  - smooth transitions
- local changes call for local updates
- local optimality does not help
- recommendations for the choice of an algorithm

Thank you for your attention!

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